

Aggregating Estimates of Firm-level Capital Distorsions

Very Preliminary – Do not circulate *

David Sraer

UC Berkeley, NBER & CEPR

David Thesmar

MIT & CEPR

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Abstract

This paper develops a framework to compute aggregate counterfactuals based on microeconomic estimates of capital distortions, which does not require the estimation of a structural model of firm dynamics. Our analysis starts from a standard dynamic general equilibrium model with heterogeneous firms subject to generic forms of capital frictions: adjustment costs, taxes and financing constraints. Within this model, we consider a randomized control trial that changes the set of distortions faced by a random subset of firms. An empiricist observing the firm-level response to this “treatment” wish to assess the effect of generalizing this treatment to all firms in the economy. We derive simple formulas to that effect. These formulas are based on sufficient statistics that can be estimated in the empirical experiment. Importantly, we provide sufficient conditions on the underlying firm dynamics model under which these formulas apply exactly and show that these conditions hold in a generic class of macro-finance models. We use simulation exercises to assess the quantitative performance of our methodology when these conditions are not satisfied.

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1 Introduction

Much progress has been made in the identification of the effect of capital distortions on firm-level outcomes. First, a large finance literature has shown that financial constraints affect firm behavior. For instance, [Lamont \(1997\)](#) and [Rauh \(2006\)](#) investigate the behavioral response of firms to cash-flow shocks; [Gan \(2007\)](#) and [Chaney et al. \(2012\)](#) use variations in local real estate prices as collateral shocks. In the earlier finance and growth literature, [Rajan and Zingales \(1998\)](#) and [Bertrand et al. \(2007\)](#) are examples of empirical studies of the impact of the structure of financial system on corporate behavior. Second, another literature in public finance focuses on firm responses to taxes. Recently, [Zwick and Mahon \(2015\)](#) find evidence that firms react to a temporary reduction in corporate taxes. [Giroud and Rauh \(2016\)](#) precisely measure the extent to which firms cut employment and investment as a response to corporate taxes. While these papers convincingly reject the null hypothesis that firms do not react to changes in capital distortions (either financing constraints or taxes), they are typically silent about the aggregate implication of the micro estimate – with the notable exception of [Larrain and Stumpner \(2017\)](#). In this paper, we offer simple formulas to aggregate the well-identified effects of capital distortions into aggregate outcomes (TFP, output). We determine the conditions under which these formulas apply, and investigate their robustness.

The firm-level response of investment and hiring to changes in firm-level capital distortions is likely to differ at the micro- and the macro-economic level. First, standard general equilibrium (GE) effects will typically dampen the macroeconomic response to firm-level shocks: for instance, if all firms receive extra financing capacity labor demand increases, raising the equilibrium wage and mitigating the initial increase in capital and labor demand. Second, changes in financing capacity reallocate inputs across firms: as distortions are reduced, capital and labor flow from firms with low marginal productivity to firms with high marginal productivity. This leads to an increase in aggregate TFP. To account for such equilibrium effects and map micro-economic estimates into macro-economic outcomes, we proceed in three steps. First, we start from a simple general equilibrium (GE) model with heterogeneous firms who face stochastic productivity shocks are subject to flexible forms of capital distortions: adjustment costs, taxes and financing frictions. We characterize the equilibrium of the economy as a function of the joint distribution of firm-level productivity and firm-level marginal revenue product of capital (MRPK). Second, we introduce within this model an experiment in which a randomly-selected subset of firms receive a treatment, which changes the capital frictions these firms face. We show how this experiment can be

used to causally assess the effect of the treatment on the joint distribution of productivity and MRPK. Third, we inject this inference on the change in the joint distribution of productivity and MRPK into our macroeconomic model to quantify how aggregate output and aggregate efficiency (TFP) would be affected if all firms received this treatment. This *aggregate counterfactual* can be computed through simple closed-form formulas that combine these estimated sufficient statistics with the model’s structural parameters.

We consider the following economic setting. A continuum of heterogeneous firms dynamically combine labor and capital to produce imperfectly substitutable inputs. These producers compete on the product and labor markets (Dixit and Stiglitz (1977)), and face three different capital frictions: adjustment costs of capital, taxes and financing constraints. The model allows for a fairly large class of such frictions. Labor is supplied elastically by households. There is no aggregate uncertainty. In the spirit of the misallocation literature (Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Hopenhayn (2014)), we rewrite firms’ investment decision as a function of the wedge between the effective marginal productivity of capital and the actual user cost of capital. This wedge is endogenous and arises because of the frictions the firm is facing in our model. In equilibrium, macro-economic outcomes such as aggregate TFP or aggregate output depend on the firm-level distribution of wedges, as well as on the correlation of these wedges with firms’ productivity shocks.

Within this economic setting, we assume that the data provide us with a micro treatment, which affects capital frictions at the firm-level. The data allow us to measure firm behavioral responses to the micro treatment, and in particular the moments of the distribution of wedges. We then aggregate this micro treatment, in the sense that we hypothetically apply the same micro treatment to all firms in the economy. To do this, we use the estimated effect of the treatment on the distribution of the wedges on treated firms, and apply it to the whole economy. An important property that is required is that the distribution of wedges is invariant when one moves from the micro treatment to its extension to the entire economy. We show that this property holds for a large class of financing constraints, tax schedules, and adjustment cost functions. It holds with imperfect or perfect competition and with constant or decreasing returns to scale, as long as frictions are homogeneous of degree 1.

An important assumption is the Cobb-Douglas production technology, which ensures that the ratio of sales to capital is equal to the marginal revenue product of capital (MRPK). However, we can relax this assumption and derive alternative formulas when the production function is CES, provided the elasticity of substitution between labor and capital is

small enough. Another important assumption is that log-MRPK is normally distributed, or, alternatively, that it has small deviations around its mean.¹ As a result of this assumption, the macro-economic implications of treating every firm in the economy can be simply derived from three sufficient statistics: (1) the effect of the treatment on average log-MRPK (2) the treatment effect on the variance of log-MRPK (3) the treatment effect on the covariance of log-MRPK and log sales. Intuitively, the first moment characterizes how the treatment causally affects the average efficiency of firms investment decisions and is similar to a level effect. The second moment characterizes how the treatment distorts the allocation of inputs across heterogeneous producers: if the treatment leads to a decrease in the variance of the log-MRPK, then treating all firms leads to a reduction in misallocation and an increase in aggregate TFP, along the lines of [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#). Finally, the third moment is important for output: if the treatment reduces the covariance of firms' capital wedge and productivity, then applying the treatment to all firms in the population will allow the average firm to produce more, since it will be able to better exploit its investment opportunities.

Macroeconomic effects can then simply be obtained by combining these three sufficient statistics with parameters of the model and we provide simple formulas to that effect. These parameters govern the key features of the macroeconomic model. Because our baseline model is simple, the aggregation only requires three parameters: (1) the capital share: the aggregate implications of firm-level distortions will be greater when the distorted input has a larger share in production; (2) the elasticity of labor supply: a reduction in firm-level distortions increases firm-level labor demand – how this firm-level effect translates into aggregate output depend on how wages adjust in equilibrium and hence on the labor supply elasticity; and most importantly (3) the degree of substitutability between inputs producers: when competition is fierce, removing distortions leads to massive reallocation of inputs across firms, which makes the behavioral response to the micro treatment much larger than its macro implication. A big benefit of our methodology is that we do not need to estimate a structural model of firm dynamics, even though firms do a potentially complex dynamic model of investment. A drawback is that we can only aggregate treatments whose effect we observe in a subset of the data.

We then check the robustness of our formula to (1) changes in the macroeconomic framework and (2) changes in key assumptions. We first investigate deviations to changes in the macroeconomic framework. First, we extend our model to a setting where input suppliers

¹Using financial statements from private firms across several countries reported in AMADEUS, we show that this is a reasonable approximation in firm-level data.

imperfectly compete within industries, and industry outputs are combined into final goods using a Cobb-Douglas technology with heterogeneous shares. Second, we perform a simulation exercise to investigate the robustness of our methodology to the assumption that wedges are log-normally distributed or have small deviations around the mean. To do this, we simulate a particular case of our dynamic model where firms face financing constraints, but no taxes nor adjustment costs. In this world, we know that aggregate output and TFP, and can evaluate the validity of our aggregation formulas. We show that, in our exercise, this assumption does not have a material effect on the validity of our formulas.

Finally, we apply our analysis to actual data. [Chaney et al. \(2012\)](#) measures the sensitivity of firm investment to collateral values, by exploiting variations in local real estate prices as shocks to the value of real estate collateral for firms owning land. We use [Chaney et al. \(2012\)](#)'s identification strategy and consider as a "treatment" the increase in collateral value triggered by the increase in local house prices over the 2000-2006 period in MSAs that experienced the largest price increase (i.e. MSAs in the top third of the land-supply elasticity distribution). The methodology developed in this paper allows us to compute the counterfactual increase in aggregate output and TFP that would have resulted if all MSAs in the US had experienced a similar house price increase. To compute this macro-economic response, we simply estimate the average effect of this "treatment" on (i) firms' average log-MRPK (ii) on the dispersion of firms' log-MRPK and (iii) the correlation of log-MRPK and firm-level output. As in [Chaney et al. \(2012\)](#), these statistics are estimated by comparing, within a given MSA, the average response of firms owning their headquarters relative to firms renting their real estate assets and then comparing this relative response across MSAs with high vs. low house price growth over the 2000-2006 period. This analysis suggests that if all firms in the US had been exposed to a rise in house prices equal to that experienced in areas in the bottom third of the land-supply elasticity distribution (i.e. high house price growth region), aggregate TFP would have increased by about 7% and aggregate output by 28%.

Related Literature

Our paper is related to three strands of the literature.

First, a long standing literature in corporate finance has been interested in measuring the effects of financing frictions on firm-level outcomes. This literature mostly started with simple investment / cash-flow regressions (e.g., [Fazzari et al. \(1988\)](#)). However, investment / cash-flow regressions have been challenged on several grounds: measurement error in Q

may bias the estimates ([Erickson and Whited \(2000\)](#)); the ex ante classification of firms in terms of credit constraint may be problematic ([Kaplan and Zingales \(1997\)](#)). As a result, the literature moved on to search for exogenous shocks to firm's funding capacity. [Lamont \(1997\)](#) uses variations in oil prices as exogenous shocks to the cash-flows available for non-oil divisions of oil conglomerates; [Rauh \(2006\)](#) exploits the mandatory contribution to defined benefit plans for firms whose pension funds are underfunded; [Gan \(2007\)](#) and [Chaney et al. \(2012\)](#) use variations in local real estate prices as shocks to the value of collateral available to land-holding firms. [Faulkender and Petersen \(2012\)](#) use the American Jobs Creation Act (AJCA) as a temporary shock to the cost of internal financing to identify the role of capital constraints in firms' investment decisions. [Lemmon and Roberts \(2010\)](#) examine how shocks to the supply of credit impact investment using the collapse of Drexel Burnham Lambert, the passage of the Financial Institutions Reform, Recovery, and Enforcement Act of 1989, and regulatory changes in the insurance industry as an exogenous contraction in the supply of below-investment-grade credit after 1989. [Banerjee and Duflo \(2014\)](#) uses variation in access to a targeted lending program in India to estimate whether firms are credit constrained. [Zia \(2008\)](#) exploits an exogenous shock to the supply of subsidized credit to exporting firms in Pakistan to identify the effect of credit constraints on firms' exporting decisions. This literature, however, remains confined to micro-economic estimates: the effect of credit constraints are estimated at the firm-level, but there is no assessment of how these estimates translate into aggregate effects. In our simple framework, these empirical settings can be used to measure sufficient statistics that are then used to compute aggregate effects.

Second, a recent literature in macroeconomics quantifies the effect of financing frictions on macroeconomic outcomes, mostly through calibration exercises. [Midrigan and Xu \(2014\)](#) calibrate a dynamic model of investment under collateral constraint with firm entry and calibrate it to match firm-level moments estimated on a sample of Korean data. [Buera et al. \(2011\)](#) develop a quantitative framework to explain the relationship between aggregate TFP and financial development. [Buera et al. \(2015\)](#) develop a tractable model with frictions in both credit and labor markets to study the aggregate implications of a sudden tightening of collateral constraints. [Gopinath et al. \(2015\)](#) analyze the macroeconomic effect on European firms of the decline in the real interest rate in the early 2000s in the context of a macroeconomic model with collateral constraints. Our model builds on [Restuccia and Rogerson \(2008\)](#), [Hsieh and Klenow \(2009\)](#) and [Hopenhayn \(2014\)](#) would model firm distortions using exogenous wedges. [Hsieh and Klenow \(2009\)](#) evaluate the

TFP losses due to capital constraints using the dispersion MRPKs. They do not, however, consider the link between a well-identified microeconomic treatment and the aggregate effect of generalizing it, nor do they derive sufficient statistics for aggregate output. Closest to our work is [Larrain and Stumpner \(2017\)](#), which use the [Hsieh and Klenow \(2009\)](#) apparatus to investigate the impact of capital account deregulation in Europe on TFP. Our contribution is more methodological: We show under which condition the application of the [Hsieh and Klenow \(2009\)](#) is valid, derive additional formulas for output, and check their theoretical robustness. But in contrast with most of the macro literature and more in line with [Larrain and Stumpner \(2017\)](#), we start from well-identified moments at the firm-level and aggregate these moments through a macroeconomic model.

Third, our goal in this paper is similar in spirit to the nascent literature seeking to account for equilibrium effects in natural experiment settings. [Crépon et al. \(2013\)](#) provide a thorough evaluation of a job placement assistance program and shows that the programs negatively affects workers not participating to the program and competing on the labor market with workers receiving assistance. [Lalive et al. \(2015\)](#) show how an extension of the duration of unemployment insurance benefits for a large group of eligible workers in selected regions of Austria positively affected non-eligible workers. [Hombert et al. \(2014\)](#) shows how a reform to the French unemployment insurance system led to a massive increase in new business registrations but affected negatively incumbents operating in industries most affected by the reform. Our modeling approach allows for such crowding out effects through endogenous variations in prices. The difference between this literature and our paper is that we do not identify these effect from the data, but using a model. In this sense, our approach is reduced form at the firm-level, but structural at the macro-level.

The rest of the paper is structured as follows. Section 2 present the economic model we used to develop our methodology. Section 3 shows what firm-level inference need to be made out of a valid empirical setting to compute the aggregate elasticity. Section 4 discusses various extensions to the model as well as alternative sufficient statistics to derive aggregate effects.

2 The Economic Model

2.1 Set-up

The economy is dynamic ($t = 0, 1, \dots, \infty$) and there is no aggregate uncertainty. We first consider a simple market structure and extend the analysis to include heterogenous industries in Section 4. A continuum of monopolists produce imperfectly substitutable intermediate goods in quantity y_i at a price p_i (Dixit and Stiglitz (1977)). There is a perfectly competitive final good market, which aggregates intermediate output according to a CES technology:

$$Y_t = \left(\int_i y_{it}^\theta di \right)^{\frac{1}{\theta}}, \quad (1)$$

We use the final good as the numeraire. Profit maximization in the final good market implies that the demand for product i is given by: $p_{it} = \left(\frac{Y_t}{y_{it}} \right)^{1-\theta}$ and $-\frac{1}{1-\theta}$ is the price elasticity of demand.

To produce y_{it} , firms combine labor and capital according to a Cobb-Douglas production function: $y_{it} = e^{z_{it}} k_{it}^\alpha l_{it}^{1-\alpha}$, where k_{it} is firm i 's capital stock in period t , l_{it} is the labor input in period t , α is the capital share and z_{it} is firm i 's idiosyncratic productivity shock in period t . With monopolistic competition and the demand system in Equation 1, firm i output in period t is $p_{it} y_{it} = Y_t^{1-\theta} y_{it}^\theta$. We assume that there is no adjustment costs to labor so that labor is a static input. If w_t be the equilibrium wage in period t faced by firms, static labor optimization implies that in period t , firm i 's profit becomes:

$$\pi_{it} = p_{it} y_{it} - w_t l_{it} = \frac{\alpha}{\alpha + (1-\alpha)\phi} \left(\frac{(1-\alpha)\phi}{\alpha + (1-\alpha)\phi} \right)^{\frac{1-\alpha}{\alpha}\phi} \left(\frac{Y_t}{w_t^{\frac{1-\alpha}{\alpha} \frac{\phi}{1-\phi}}} \right)^{1-\phi} e^{\frac{\phi}{\alpha} z_{it}} k_{it}^\phi,$$

where $\phi = \frac{\alpha\theta}{1-(1-\alpha)\theta} < 1$. Productivity shocks (z_{it}) are markovian and $H(z_{it+1}|z_{it})$ is the c.d.f of z_{it+1} conditional on z_{it} .

The capital good consists of final good – so that its price is also 1 – and it depreciates at a rate δ . Capital investment in period t is subject to a one period time-to-build. Firms can finance investment using the profits they realize from operations or through external financing. The first source of outside financing is debt. b_{it+1} is the total real payment due to creditors in period $t + 1$. To simplify notations, we define $x_{it} = (k_{it}, k_{it+1}, b_{it}, b_{it+1})$ and we

introduce Θ , a vector containing all the model's parameters. $r_{it} = r(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)$ is the interest rate on the loan granted at date t , so that $\frac{b_{it+1}}{1+r_{it}}$ is the proceed from debt financing received in period t . We allow the firm's investment and debt financing at date t to be subject to adjustment costs $\Gamma(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)$. We also assume that firms pay taxes and receive subsidies: $\mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)$ corresponds to the net tax paid by the firm.

The second source of outside funding is equity. The firm can raise funds from shareholders in the equity market, or distribute excess funds to shareholders: e_{it} is the equity issuance (if negative) or distribution (if positive) made by firm i in period t : it corresponds to the financing gap left after all other sources of financing have been used:

$$\begin{aligned} e_{it} &= \pi_{it} - (k_{t+1} - (1 - \delta)k_t) - \Gamma(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) + \left(\frac{b_{it+1}}{1 + r_{it}} - b_{it} \right) - \mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) \\ &= e(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) \end{aligned}$$

We consider generic financing frictions. First, equity issuance may be costly, and we note $\mathcal{C}(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)$ these equity issuance costs. Second, the amount of outside financing may be constrained, a friction that we capture through a vector of constraint: $M(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) \leq 0$.

The timing is standard in models of firm dynamics. At the beginning of period t , productivity z_{it} is realized. The firm then combines capital in place k_{it} with labor l_{it} to produce and receive the corresponding profits. It then selects the next period stock of capital k_{it+1} , pays the corresponding adjustment costs, reimburse its existing debt b_{it} and receive the proceed from debt issuance $\frac{b_{it+1}}{1+r_{it}}$.

Omitting the it index and denoting with prime next-period variables, we can represent firms optimization problem through the following Bellman equation:

$$V(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \beta \mathbb{E}_{z'} [V(z', k', b'; \Theta, w', Y') | z], \\ M(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases} \quad (2)$$

where β^F is the firm's discount rate. In what follows, we assume the standard conditions on the cost functions and the constraints so that the contraction mapping theorem applies to this Bellman equation and there is a unique value function.

The household side of the economy is straightforward. A representative household has

linear preferences over consumption and leisure: $u(c_t, l_t) = c_t - \frac{l_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \frac{\bar{w}}{\bar{L}^{\frac{1}{\epsilon}}}$, where c_t is period t consumption, l_t is period t labor supply, ϵ is the Frisch elasticity and (\bar{w}, \bar{L}) are constant. The representative household owns all firms in the economy. w_t is the equilibrium wage in period t . β^H is the representative household's discount rate. In the absence of aggregate uncertainty, optimal consumption and labor supply decisions imply that $l_t^s = \bar{L} \left(\frac{w}{\bar{w}}\right)^\epsilon$ and $\beta^H = \frac{1}{1+r_f}$, where r_f is the exogenous risk-free rate. Note that since households portfolios are well diversified across firms, we also have $\beta^F = \frac{1}{1+r_f}$ even though the model potentially allows for firms' default.

2.2 The distribution of capital wedges

Instead of solving the model explicitly, we will characterize its equilibrium as a function of the distribution of objects defined as capital wedges τ_{it} . These wedges are defined as the ratio of a firm's marginal revenue product of capital to the user cost of capital $r_f + \delta$ for firm i in period t . They measure how much firms' capital stock deviates from static and frictionless optimization. In our model, firms potentially deviate from the static, frictionless optimum for three reasons: financing frictions, adjustment costs, and taxes.

Definition 1 (Definition of wedges).

$$1 + \tau_{it} = 1 + \tau(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) = \frac{1}{r_f + \delta} \frac{\partial p_{it} y_{it}}{\partial k_{it}} = \frac{\alpha \theta}{r_f + \delta} \frac{p_{it} y_{it}}{k_{it}}$$

We now show that under certain assumptions, the distribution of these wedges *in the steady-state* does not depend on the equilibrium quantities w and Y . As we detail below, the assumptions necessary to obtain this result are satisfied in a large class of macro-finance models.

Proposition 1 (Distribution of wedges).

Assume that the adjustment cost function $\Gamma()$, the tax function $\mathcal{T}()$, the vector of funding constraint function $M()$ and the equity issuance cost function $C()$ satisfy the following property:

$$\forall (z, \mathbf{x}; \Theta, w, Y), \quad F(z, \mathbf{x}; \Theta, w, Y) = S \times F\left(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1\right), \quad \text{where: } S = \frac{Y}{w^{\frac{(1-\alpha)\phi}{(1-\phi)\alpha}}} \quad (3)$$

Assume also that the interest rate function $r(\cdot)$ satisfies the following property:

$$\forall (z, \mathbf{x}; \Theta, w, Y), \quad r(z, \mathbf{x}; \Theta, w, Y) = r\left(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1\right)$$

Then, at the steady-state of the economy, the c.d.f. of the wedge distribution τ_i does not depend on (w, Y) and depends only on the vector of model parameters Θ : $G(\tau_i; \Theta)$.

Similarly, the c.d.f. of the joint distribution of capital wedges τ_i and productivity z_i is independent of (w, Y) : $F(z_i, \tau_i; \Theta)$.

Proof. See Appendix [B.1](#) □

2.3 Correspondence with standard models of firm dynamics

The assumption under which Proposition [1](#) hold may appear to be restrictive. However, they encompass a large set of standard models used in the literature.

Adjustment Costs

Consider first the case of adjustment costs. Quadratic adjustment costs to capital, linear adjustment costs of capital, fixed costs of adjustments that scale either with production, output and capital or discount for capital resale all satisfy the assumptions in Proposition [1](#). For instance, if $\Gamma(\cdot)$ is given by:

$$\Gamma(z, \mathbf{x}; \Theta, w, Y) = \gamma_1 \frac{(k' - (1 - \delta)k)^2}{k} + \gamma_2 k + \mathbb{1}_{\{k' - (1 - \delta)k \neq 0\}} (\gamma_3 y + \gamma_4 py + \gamma_5 k) + \gamma_6 k \mathbb{1}_{\{k' - (1 - \delta)k < 0\}},$$

then, since $y(z, k; \Theta, w, Y) = S \times y(z, \frac{k}{S}; \Theta, 1, 1)$ and $py(z, k; \Theta, w, Y) = S \times py(z, \frac{k}{S}; \Theta, 1, 1)$, it is trivial to show that $\Gamma(z, \mathbf{x}; \Theta, w, Y) = S \times \Gamma(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$

Financing

Second, consider the financing side of the model. Our formulation encompass standard models of financing constraints and investment. For instance, in [Michaels et al. \(2016\)](#) or [Gilchrist et al. \(2014\)](#), debt is risky and in the event that the firm is unable to repay, the lender can seize a fraction $1 - \zeta$ of the firm's fixed assets k . The firms's future market value is not collateralizable, so that a firm's access to credit is mediated by a net worth

covenant, which restrains the firm's ability to sell new debt based on its current physical assets and liabilities. Concretely, default is triggered when net worth reaches 0, which defines a threshold value for productivity \hat{z} such that:

$$0 = \underbrace{\left(\frac{(1-\alpha)\phi}{\alpha + (1-\alpha)\phi} \right)^{\frac{(1-\alpha)\phi}{\alpha}}}_{=\kappa_0} S^{1-\phi} e^{\frac{\phi}{\alpha}\hat{z}} k^\phi - b + c^k(1-\delta)k, \quad (4)$$

where c_k is the second-hand price of capital, which we treat as a technological parameter. As in [Michaels et al. \(2016\)](#), the right side of the previous equation represents the resources that the firm could raise in order to repay its debt just prior to bankruptcy, which is why its capital is valued at the second-hand price c_k . The wage bill is absent from the previous equation because labor is paid in full, even if the firm subsequently defaults. Finally, the face value of debt discounted at the interest rate $r(z, \mathbf{x}; \Theta, w, Y)$ must equal the debt holder's expected payoff discounted at the risk-free rate:

$$\frac{1}{1+r_f} \left[\int_0^{\hat{z}} \left(\kappa_0 S^{1-\phi} e^{\frac{\phi}{\alpha}z'} k'^\phi + (1-\zeta)(1-\delta)k' \right) dH(z'|z) + (1-H(\hat{z}|z))b' \right] = \frac{b'}{1+r(z, \mathbf{x}; \Theta, w, Y)} \quad (5)$$

Equations 4 and 5 provides the joint definition for the interest rate function $r(z, \mathbf{x}; \Theta, w, Y)$. To complete the description of the model, we can assume, following [Michaels et al. \(2016\)](#) and [Gilchrist et al. \(2014\)](#) that equity issuances are subject to an underwriting fees such that there is a positive marginal cost to issue equity:

$$\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = \lambda |e(z, \mathbf{x}; \Theta, w, Y)| \mathbb{1}_{\{e(z, \mathbf{x}; \Theta, w, Y) < 0\}}$$

This particular model of financing satisfies the different assumptions of [Proposition 1](#). Note first that equation 4 can be rewritten as: $0 = \kappa_0 e^{\frac{\phi}{\alpha}\hat{z}} \frac{k^\phi}{S} - \frac{b}{S} + c^k(1-\delta)\frac{k}{S}$. As a result, it is clear that $\hat{z}(k, b; \Theta, w, Y) = \hat{z}(\frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)$. As a result, we can rewrite Equation 5 as:

$$\frac{1}{1+r_f} \left[\int_0^{z(\frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)} \left(\kappa_0 e^{\frac{\phi}{\alpha}z'} \frac{k'^\phi}{S} + (1-\zeta)(1-\delta)\frac{k'}{S} \right) dH(z'|z) + (1-H(\hat{z}|z))\frac{b'}{S} \right] = \frac{\frac{b'}{S}}{1+r(z, \mathbf{x}; \Theta, w, Y)},$$

so that $r(z, \mathbf{x}; \Theta, w, Y) = r(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$. Finally, given that $e(z, \mathbf{x}; \Theta, w, Y) = S e(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$, it is obvious that $\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = S \times \mathcal{C}(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1)$. Thus, the financing frictions specified in [Gilchrist et al. \(2014\)](#) and [Michaels et al. \(2016\)](#) satisfy the assumptions of [Proposition](#)

1. More generally, the specification of debt renegotiation in [Hennessy and Whited \(2007\)](#) would also satisfy these assumptions.

Another example of model encompassed in our formulation is [Midrigan and Xu \(2014\)](#). In their model, debt is assumed to be risk-free through full collateralization: $b' \leq \xi k'$ so that $r(z, \mathbf{x}; \Theta, w, Y) = r_f$ and producers can only issue claims to a fraction χ of their future profits: $e(z, \mathbf{x}; \Theta, w, Y) \geq -\chi V(z, \mathbf{x}; \Theta, w, Y)$. In this case, the vector $M(z, \mathbf{x}; \Theta, w, Y)$ consists of the last two inequalities, and it is direct to see that both M and $r(\cdot)$ satisfy the assumptions of Proposition 1. Of course, any combination of the constraints in [Midrigan and Xu \(2014\)](#) and [Hennessy and Whited \(2007\)](#) would also satisfy these assumptions.

Additionally, the following ingredients would also be consistent with Proposition 1: (1) debt financing could be limited by existing or future cash flows ($b \leq \iota e(z, \mathbf{x}; \Theta, w, Y)$) (2) there can be fixed costs of equity issuance that scales, for instance, with the value of the firm: $\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = \nu V(z, \mathbf{x}; \Theta, w, Y) \mathbb{1}_{e(z, \mathbf{x}; \Theta, w, Y) < 0}$ (3) there can be quadratic costs of equity issuance of the form: $\mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = \Psi \frac{(e(z, \mathbf{x}; \Theta, w, Y))^2}{V(z, \mathbf{x}; \Theta, w, Y)}$

Taxes

Standard specifications for the corporate income tax satisfy the assumption of Proposition 1: $\mathcal{T}(z, \mathbf{x}; \Theta, w, Y) = \tau \max(0, \pi(z, \mathbf{x}; \Theta, w, Y) - \delta k - b)$.

2.4 Equilibrium

We now solve the competitive equilibrium of this economy in the steady state as a function of the distribution of the capital wedges defined in Definition 1. In this simple version of the model, the steady state equilibrium corresponds to an equilibrium wage w that clears the labor market and aggregate output Y that clears the final good market.

Given static labor optimization and the definition of capital wedges, we can write the firm-level demand for labor and capital as:

$$\begin{cases} k_{it} = \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_{it}} \left(\frac{\alpha\theta}{(r_f + \delta)(1 + \tau_{it})} \right)^{\frac{1-(1-\alpha)\theta}{1-\theta}} \\ l_{it} = \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{1-\alpha\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_{it}} \left(\frac{\alpha\theta}{(r_f + \delta)(1 + \tau_{it})} \right)^{\frac{\alpha\theta}{1-\theta}} \end{cases} \quad (6)$$

We combine these two expressions to calculate firm-level production, which we then aggregate using the final good production function. This leads to the following expression for the equilibrium real wage:

$$1 = \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha\theta}{1-\theta}} \int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta) \quad (7)$$

Labor supply is equal to the aggregated firm-level labor demand:

$$Y = \bar{L} \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^\epsilon \left(\frac{\alpha\theta}{r_f + \delta} \right)^{(1+\epsilon)\frac{\alpha}{1-\alpha}} \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta) \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}} \quad (8)$$

Aggregate TFP is defined as $TFP = \frac{Y}{K^\alpha L^{1-\alpha}}$ (Hsieh and Klenow (2009)) and is given by:

$$TFP = \frac{\left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta) \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta) \right)^\alpha} \quad (9)$$

3 Inference and Aggregation

3.1 Binary treatment

In this section, we show how to perform aggregate counterfactuals using firm-level estimates. To simplify the exposition, we assume in this section that the empirical exercise consists of a simple binary treatment, where a random subset of firms are treated. We show in Section 3.3 how to generalize this analysis.

More precisely, a treatment here is defined as a change in one or several model parameters Θ for a random subset of treated firms: non-treated firms ($T = 0$) operate with $\Theta = \Theta_0$ while treated firms ($T = 1$) operate with $\Theta = \Theta_1$. The subset of parameters affected by the experiment can relate either to financing costs (the vector of constraint $M()$, equity issuance costs $\mathcal{C}()$ or the interest rate $r()$), to adjustment costs $\Gamma()$, or to the tax schedule $\mathcal{T}()$. Note that we exclude the case where the treatment affects the capital share in production α , the price elasticity of demand θ , the labor supply elasticity ϵ or the depreciation rate δ .

For instance, the treatment may consist in a randomized experiment on the pledgeability of tangible capital (ζ in the example of Section 2.3). It can be a subsidized lending

program, thereby affecting the risk-free rate effectively faced by treated firms (Banerjee and Duflo (2016)). It can also consist of random variations in the efficiency of debt markets, equity markets or both, which, in the model described above following Midrigan and Xu (2014), is equivalent to an increase in χ and ξ (Rajan and Zingales (1998), Jarayatne and Strahan (1996), Calomiris et al. (2017)).

Given this experimental setting, we now define the aggregate counterfactual we are interested in, which generalizes the treatment to all firms in the economy:

Objective 1 (The aggregation exercise). *The aggregate effect of a treatment is defined by the difference in aggregate outcomes (output, wages and aggregate TFP) between a counterfactual economy in which all firms receive the treatment (and thus all firms operate with $\Theta = \Theta_1$) and the economy in which no firm receives the treatment (and thus all firms operate with $\Theta = \Theta_0$).*

Formally, the aggregation exercise defined in Objective 1 consists in estimating the following quantity:

$$\begin{aligned}\Delta \ln(Y) &= \ln(Y(\Theta = \Theta_1)) - \ln(Y(\Theta = \Theta_0)) \\ &= \frac{(1 + \epsilon)(1 - \theta)}{(1 - \alpha)\theta} \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right],\end{aligned}$$

as well as:

$$\begin{aligned}\Delta \ln(TFP) &= \ln(TFP(\Theta = \Theta_1)) - \ln(TFP(\Theta = \Theta_0)) \\ &= \frac{1 - (1 - \alpha)\theta}{\theta} \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right] \\ &\quad - \alpha \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right],\end{aligned}$$

where we simply use the aggregate output formula from Equation 8 and 9.

To perform these counterfactuals, we proceed through the following steps:

1. Exploit the experiment to make inference on how the steady-state joint distribution of capital wedges and productivity is affected by the treatment, i.e estimate $\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_k)$ and $\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_k)$ for firms operating with $\Theta_k = \Theta_1$ and firms operating with $\Theta_k = \Theta_0$. If the experiment is valid, this inference can be done in an unbiased way.

2. Within our macroeconomic model, use the estimated terms in Step 1 to compute $\Delta \ln(Y)$ and $\Delta \ln(TFP)$.

Crucially, the validity of this approach relies on the property that the joint distribution of capital wedges and productivity $F(z, \tau; \Theta)$ does *not* depend on the general equilibrium quantities (w, Y) . Otherwise, when generalizing the treatment in our counterfactual economy, the steady-state (w, Y) would change, which would in turn generate changes in the steady-state distribution of capital wedges that cannot be observed in the data, preventing us to perform our counterfactual exercise. However, under the assumptions of Proposition 1, we know that the steady-state distribution of capital wedges τ_i only depends on the model's parameters and does not depend on (w, Y) , i.e. $F(z, \tau; \Theta, w, Y) = F(z, \tau; \Theta)$. As a result, as we generalize the experiment, the changes in the distribution of capital wedges observed in the experiment carry through to the counterfactual economy where all firms receive the treatment.

We now provide simple formulas to perform the aggregation exercise in Objective 1, without making any additional parametric assumptions or having to estimate firm-level TFP.

Proposition 2.

Let D_l and D_k be the following sufficient statistics estimated from the experiment in the steady-state of the economy where only a 0-measure set of firms receive the treatment (\bar{w}, Y_0) :

$$\begin{cases} D_l = \ln(\mathbb{E}[l_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[l_{it}|T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \\ D_k = \ln(\mathbb{E}[k_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[k_{it}|T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \end{cases}$$

The effect of generalizing the treatment to all firms in the economy on steady-state aggregate output and TFP is given by:

$$\begin{cases} \Delta \ln(Y) = \frac{(1 + \epsilon)(1 - \theta)}{(1 - \alpha)\theta} \times D_l \\ \Delta \ln(TFP) = \left(\frac{1}{\theta} - (1 - \alpha) \right) \times D_l - \alpha \times D_k \end{cases}$$

Proof. See Appendix B.2. □

Empirically, it may prove difficult to estimate efficiently D_l and D_k as labor and capital are often fat-tailed. For instance, labor and capital may be log-normally distributed, in

which case differences in sample means will not constitute efficient estimators of D_l and D_k . One such case arise when productivity shocks z_{it} and wedges τ_{it} are jointly log-normally distributed:

$$\begin{pmatrix} \ln(z_{it}) \\ \ln(1 + \tau_{it}) \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mu_z \\ \mu_\tau(\Theta) \end{pmatrix}, \begin{pmatrix} \sigma_z & \sigma_{\tau z}(\Theta) \\ \sigma_{\tau z}(\Theta) & \sigma_\tau^2(\Theta) \end{pmatrix} \right], \quad (10)$$

where Θ corresponds to the set of parameters under which all firms operate in the economy, and μ_τ , $\sigma_{\tau z}$ and σ_τ^2 depend only on Θ and not on (w, Y) thanks to Proposition 1.

The assumption that productivity shocks are log-normally distributed is common in the investment literature. Since $\log\left(\frac{p_i y_{it}}{k_{it}}\right) = \ln\left(\frac{r}{\alpha\theta}\right) + \ln(1 + \tau_i)$, Assumption 10 implies that the log-marginal revenue product of capital (IMRPK) is also distributed according to a normal distribution. We test the relevance of this assumption using data from BvD AMADEUS Financials for the year 2014. We follow Gopinath et al. (2015) and measure value added, $p_i y_{it}$, as the difference between gross output (operating revenue) and materials. We measure the capital stock, k_{it} , with the book value of fixed tangible and intangible assets. For each firm in this sample, we then compute the logarithm of the ratio of $p_i y_{it}$ and k_{it} . For 9 countries in our sample (France, Italy, Spain, UK, Portugal, Croatia, Sweden, Bulgaria and Romania), we report in Figure 1 normal probability plots, i.e. plots of the empirical c.d.f. of the standardized IMRPK against the c.d.f. of a normal distribution. Figure 1 shows the relevance of the parametric assumption 10 in real output data.

Under Assumptions 10, aggregate output and TFP become:

$$\begin{cases} \ln Y = A + \frac{\alpha(1+\epsilon)}{1-\alpha} \left(\mu_z - \alpha\mu_\tau(\Theta) + \frac{1}{2} \frac{\theta}{1-\theta} (\alpha^2 \sigma_\tau^2(\Theta) + \sigma_z^2 - 2\alpha\sigma_{\tau z}(\Theta)) \right) \\ \ln(TFP) = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) \sigma_\tau^2(\Theta) + \left(\mu_z + \frac{1}{2} \frac{\theta}{1-\theta} \sigma_z^2 \right) \end{cases} \quad (11)$$

where $A = \ln \left(\bar{L} \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^\epsilon \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha(1+\epsilon)}{1-\alpha}} \right)$.

These formulas illustrate forces already discussed in the literature. Dispersion in log-MRPK impairs aggregate efficiency because it creates capital misallocation (Hsieh and Klenow (2009)). A positive correlation between productivity and distortions also hurts aggregate production: output is lower when the most productive firms experience the largest distortions (Hopenhayn (2014)). However, in our setting, such a positive correlation does not affect aggregate TFP. This result emanates from the log-normality assumption. It does not hold in Restuccia and Rogerson (2008), who use a binary distribution for the distribu-

tion of distortions.

Differentiating output and TFP from equations (11), we obtain the output and TFP changes of moving from an economy where all firms have parameters Θ_0 to an economy where all firms have parameters Θ_1 :

$$\begin{cases} \Delta \ln Y = \frac{\alpha(1+\epsilon)}{1-\alpha} \left[-\Delta\mu_\tau + \frac{1}{2} \frac{\theta}{1-\theta} (\alpha\Delta\sigma_\tau^2 - 2\Delta\sigma_{\tau z}) \right] \\ \Delta \ln TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) \Delta\sigma_\tau^2 \end{cases} \quad (12)$$

where $\Delta\mu_\tau = \mu_\tau(\Theta = \Theta_1) - \mu_\tau(\Theta = \Theta_0)$, $\Delta\sigma_\tau^2 = \sigma_\tau^2(\Theta = \Theta_1) - \sigma_\tau^2(\Theta = \Theta_0)$ and $\Delta\sigma_{\tau z} = \sigma_{\tau z}(\Theta = \Theta_1) - \sigma_{\tau z}(\Theta = \Theta_0)$

We now provide alternative formulas under Assumptions 10 that allow us to perform the aggregation exercise in Objective 1 using sufficient statistics that are more likely to be empirically well-behaved than the sufficient statistics D_l and D_k used in Proposition 2:

Proposition 3.

Let D_{lMRPK} , DV_{lMRPK} and $DCOV$ be the estimated firm-level effect of the treatment on the average log-MRPK, on the dispersion of log-MRPK and on the covariance of log-output and log-MRPK measured in the steady of the economy where only a 0-measure set of firms receive the treatment (\bar{w}, Y_0) :

$$\begin{cases} D_{lMRPK} = \mathbb{E} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0) \right] - \mathbb{E} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0) \right] \\ DV_{lMRPK} = \mathbb{V}ar \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0) \right] - \mathbb{V}ar \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right) | T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0) \right] \\ DCOV = \mathbb{C}ov \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0) \right] - \mathbb{C}ov \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0) \right] \end{cases}$$

The effect of generalizing the treatment to all firms in the economy on aggregate output and TFP is given by:

$$\begin{cases} \Delta \ln Y = -(1+\epsilon) \frac{\alpha}{1-\alpha} \left(D_{lMRPK} + \frac{1}{2} \frac{\alpha\theta}{1-\theta} DV_{lMRPK} + DCOV \right) \\ \Delta \ln TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) DV_{lMRPK} \end{cases}$$

Proof. See Appendix B.3. □

The intuitions for the expressions in Proposition 3 are direct. First, each of the three sufficient statistics derived above and estimated at the firm-level is important in estimating the aggregate counterfactual. If the treatment leads to a causal reduction in the average firm-level log-MRPK, this implies that on average, firm-level production increases after receiving the treatment, which, at the aggregate level, should lead to an increase in total output. If the treatment leads to a decrease in the dispersion of log-MRPK, then, as in Hsieh and Klenow (2009), the treatment leads to a decrease in the *misallocation* of capital among treated firms, so that generalizing the treatment would lead to an increase in aggregate TFP, and therefore, all else equal, an increase in aggregate output. Finally, if the treatment leads to a causal decrease in the covariance of firm-level log output and log-MRPK, then the treatment allows the most productive firms to become less constrained and thus leads to an increase in total output.

Proposition 3 also highlights that the effect of each of these sufficient statistics on aggregate output increases with α and ϵ . α is the share of the distorted input in the production function; for given firm-level inefficiencies, a larger capital share implies larger aggregate effects. ϵ is the aggregate labor supply elasticity; when it is low, a treatment reducing inefficiencies at the firm-level will lead to large wage increases, which will dampen the effect of the treatment on aggregate output. Finally, an increase in the dispersion of log-MRPK of treated firms will affect aggregate efficiency and output all the more that there is intense competition among producers of intermediate goods (i.e. θ is larger): as θ increases, the distribution of output among producers become more concentrated, which magnifies the efficiency losses arising from the capital constraint.

3.2 Generalization: large experiments

In Section 3.1, we derived our results in the context of a small experiment, i.e. an experiment with a zero-measure set of treated firms. Our results easily extend to the case where a fraction ϑ of the population receives (randomly) the treatment. In this case, two aggregate counterfactuals can be computed:

1. Changes in aggregate outcomes starting from a *counterfactual* economy where no firms receive the treatment to another *counterfactual* economy where all firms receive the treatment
2. Changes in aggregate outcomes starting from the *actual* economy (where a random fraction μ of firms receive the treatment) to a *counterfactual* economy where all firms

receive the treatment.

Proposition 4.

Changes in aggregate output and TFP in Counterfactual 1 are given by the formulas in Proposition 2 and 3.

Changes in aggregate output and TFP in Counterfactual 2 are given by:

$$\begin{cases} \Delta \ln(Y) = -\frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \ln\left(\vartheta + (1-\vartheta)e^{\frac{\alpha\theta}{1-\theta}(D_{IMRPK} + \frac{1}{2}\frac{\alpha\theta}{1-\theta}DV_{IMRPK} + DCOV)}\right) \\ \Delta \ln(TFP) = -\ln\left(\frac{\left(\vartheta + (1-\vartheta)e^{\frac{\alpha\theta}{1-\theta}(D_{IMRPK} + \frac{1}{2}\frac{\alpha\theta}{1-\theta}DV_{IMRPK} + DCOV)}\right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\vartheta + (1-\vartheta)e^{\frac{1-(1-\alpha)\theta}{1-\theta}(D_{IMRPK} + DCOV + \frac{\alpha\theta-(1-\theta)}{2(1-\theta)}DV_{IMRPK})}\right)^\alpha}\right) \end{cases}$$

Proof. See Appendix B.4. □

3.3 Generalization: continuous treatment and elasticities

In the previous section, we considered for expositional purposes the case of a binary treatment. Our results easily extend to a setting where there is a large number of treatments, or even a continuum of treatments: $T_l \in \Omega$, where Ω is the support of all possible treatments and a treatment is defined by a particular combination of model parameters $\Theta = \Theta_l$. In this case, one possible aggregation exercise consists of applying treatment T_l to all firms in the economy, relative to a benchmark Θ_0 . To this end, we simply extend the definition of the sufficient statistics defined in Proposition 3:

$$\begin{cases} D_{IMRPK}(T_l) = \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \middle| T_{it} = T_l, \Theta_{it} = \Theta_l; (\bar{w}, Y_0)\right] - \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \middle| T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)\right] \\ DV_{IMRPK}(T_l) = \mathbb{V}\text{ar}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \middle| T_{it} = T_l, \Theta_{it} = \Theta_l; (\bar{w}, Y_0)\right] - \mathbb{V}\text{ar}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \middle| T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)\right] \\ DCOV(T_l) = \text{Cov}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right), \ln(p_{it}y_{it}) \middle| T_{it} = T_l, \Theta_{it} = \Theta_l; (\bar{w}, Y_0)\right] - \text{Cov}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right), \ln(p_{it}y_{it}) \middle| T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)\right] \end{cases}$$

We then obtain a similar aggregation formula as in Section 3.1

Proposition 5. *Relative to an economy where all firms operate under $\Theta = \Theta_0$, an economy where all firms receive treatment $T_l \in \Omega$ will experience a change in aggregate output of:*

$$\Delta \ln Y = -(1+\epsilon)\frac{\alpha}{1-\alpha} \left(D_{IMRPK}(T_l) + \frac{1}{2}\frac{\alpha\theta}{1-\theta}DV_{IMRPK}(T_l) + DCOV_{IMRPK,IR}(T_l) \right), \quad (13)$$

as well as a change in aggregate TFP of:

$$\Delta \ln TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) DV_{LMRPK}(T_l) \quad (14)$$

Proof. The proof is identical to the proof of Proposition 3. □

4 Robustness

This section discusses the robustness of our approach to assumptions about the production function or the market structure.

4.1 Heterogeneous industries

In this section, we consider a more general model than the one presented in Section 2: the economy features industries that are heterogeneous in (1) their output share in total output (2) their labor share (3) the degree of competition between firms within the industry (4) the parameters that govern the firm-level dynamics of investment and hiring and (5) potentially the treatment they receive.

More precisely, let S be the number of industries and M_s the set of firms operating in industry s . Firms in each industry produce in monopolistic competition as in Section 2, and the price-elasticity of demand $-\frac{1}{1-\theta_s}$ can be industry specific: $Y_s = \left(\int_i y_{is}^{\theta_s} \right)^{\frac{1}{\theta_s}}$. The final good market produces by combining each industry output according to the following Cobb-Douglas production function:

$$\ln(Y) = \sum_{k=1}^S \phi_s \ln(Y_s) \quad \text{and} \quad \sum_{s=1}^S \phi_s = 1. \quad (15)$$

Within industry s , the production function is given by: $y_{it} = e^{z_{it}} k_{it}^{\alpha_s} l_{it}^{1-\alpha_s}$, i.e. the labor share is assumed to be industry-specific, while we assume that the distribution of idiosyncratic productivity shocks is common across industries. Beyond ϕ_s , α_s and θ_s , an industry is also characterized by the vector of parameters that govern the firm-level optimization problem 2: Θ_s is the set of parameters under which firms in industry $s \in [1, S]$ operate. Finally, a treatment in this economy corresponds to these parameters going from Θ_s^0 to Θ_s^1 for a zero-measure randomly selected set of firms in each industry.

The following proposition shows how extending treatments to all firms in the economy would affect aggregate output and TFP:

Proposition 6. *Under this alternative market structure and under the assumption of Proposition 1, Proposition 1 remains valid: at the steady-state of the economy, the c.d.f. of the joint distribution of capital wedges τ_i and productivity z_i is independent of w , Y and the vector of industry-level output $(Y_s)_s$ and only depend on Θ_s – the set of parameters governing the firm-level optimization problem in industry s – as well as the structural parameters $(\theta_s, \alpha_s, \phi_s)$. We note this c.d.f.: $F_s(z_i, \tau_i; \Theta_s)$.*

Define the following statistics estimated from the experiment: $D_l(s) = \mathbb{E}[l_{it}|T_{it} = 1, \Theta_{it}^s = \Theta_1^s, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E}[l_{it}|T_{it} = 0, \Theta_{it}^s = \Theta_0^s, s_i = s; (w, (Y_s)_s, Y)]$ and $D_k(s) = \mathbb{E}[k_{it}|T_{it} = 1, \Theta_{it}^s = \Theta_1^s, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E}[k_{it}|T_{it} = 0, \Theta_{it}^s = \Theta_0^s, s_i = s; (w, (Y_s)_s, Y)]$. Then extending the treatment to all firms in the economy would imply a change in aggregate output of:

$$\Delta \ln Y = (1 + \epsilon) \sum_{s=1}^S \left(\frac{\phi_s(1 - \theta_s)}{\theta_s \sum_{l=1}^S (1 - \alpha_l)\phi_l} \right) D_l(s),$$

and to a change in aggregate TFP of:

$$\Delta \ln(TFP) = \sum_{s=1}^S \phi_s \frac{1 - \theta_s}{\theta_s} D_l(s) - \left(\sum_{s=1}^S \alpha_s \phi_s \right) \ln \left(\sum_{s=1}^S \frac{K_s}{K} e^{(D_k(s) - D_l(s))} \right),$$

where $\frac{K_s}{K}$ is the share of industry s in the aggregate capital stock.

If we make the additional parametric assumption that, within an industry s , the joint distribution of productivity and capital wedges is log-normally distributed:

$$\forall i \in M_s, \quad \begin{pmatrix} \ln(z_{it}) \\ \ln(1 + \tau_{it}) \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mu_z \\ \mu_\tau^s(\Theta) \end{pmatrix}, \begin{pmatrix} \sigma_z & \sigma_{\tau z}^s(\Theta) \\ \sigma_{\tau z}^s(\Theta) & (\sigma_\tau^s(\Theta))^2 \end{pmatrix} \right], \quad (16)$$

then the change in aggregate output in the counterfactual economy is given by:

$$\Delta \ln Y = -(1 + \epsilon) \sum_{s=1}^S \left(\frac{\alpha_s \phi_s \theta_s}{\theta_s \sum_{l=1}^S (1 - \alpha_l) \phi_l} \right) \left(D_{IMRPK}(s) + \frac{1}{2} \frac{\alpha_s \theta_s}{1 - \theta_s} DV_{IMRPK}(s) + DCOV(s) \right),$$

and the change in aggregate TFP is given by:

$$\begin{aligned} \Delta \ln(TFP) &= - \sum_{s=1}^S \alpha_s \phi_s \left(D_{IMRPK}(s) + \frac{1}{2} \frac{\alpha_s \theta_s}{1 - \theta_s} DV_{IMRPK}(s) + DCOV(s) \right) \\ &\quad - \left(\sum_{s=1}^S \alpha_s \phi_s \right) \ln \left(\sum_{s=1}^S \frac{K_s}{K} e^{-D_{IMRPK}(s) + \frac{1}{2} DV_{IMRPK}(s) - DCOV(s)} \right), \end{aligned}$$

where the sufficient statistics are defined as:

$$\begin{cases} D_{IMRPK}(s) = \mathbb{E} \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_{it} = 1, \Theta_{it} = \Theta_s^1, s_i = s; (\bar{w}, Y_0) \right] - \mathbb{E} \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_{it} = 0, \Theta_{it} = \Theta_s^0, s_i = s; (\bar{w}, Y_0) \right] \\ DV_{IMRPK}(s) = \mathbb{V}ar \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_{it} = 1, \Theta_{it} = \Theta_s^1, s_i = s; (\bar{w}, Y_0) \right] - \mathbb{V}ar \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right) | T_{it} = 0, \Theta_{it} = \Theta_s^0, s_i = s; (\bar{w}, Y_0) \right] \\ DCOV(s) = \mathbb{C}ov \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right), \ln(p_{it} y_{it}) | T_{it} = 1, \Theta_{it} = \Theta_s^1, s_i = s; (\bar{w}, Y_0) \right] \\ \quad - \mathbb{C}ov \left[\ln \left(\frac{p_{it} y_{it}}{k_{it}} \right), \ln(p_{it} y_{it}) | T_{it} = 0, \Theta_{it} = \Theta_s^0, s_i = s; (\bar{w}, Y_0) \right] \end{cases}$$

If we additionally assume that the sufficient statistics $D_{IMRPK}(s)$, $DV_{IMRPK}(s)$ and $DCOV(s)$ are small (i.e. $|\Theta_s^1 - \Theta_s^0| < 1$), then the aggregate effect of generalizing the treatments to all firms in the economy on aggregate TFP is, to a first-order approximation, equal to:

$$\begin{aligned} \Delta \ln(TFP) &= - \left(\sum_{l=1}^S \alpha_l \phi_l \right) \left(\sum_{s=1}^S \left(\frac{\alpha_s \phi_s}{\sum_{l=1}^S \alpha_l \phi_l} - \frac{K_s}{K} \right) (D_{IMRPK}(s) + DCOV(s)) \right) \\ &\quad + \sum_{s=1}^S \left(\frac{\alpha_s^2 \phi_s \theta_s}{\sum_{l=1}^S \alpha_l \phi_l} + \frac{K_s}{K} \right) \frac{DV_{IMRPK}(s)}{2} \end{aligned}$$

Proof. See Appendix B.5. □

If the experiment is run only on a subset of industries, then our methodology allows to compute changes in aggregate output and TFP in the counterfactual economy where

all firms *in these industries* receive the treatment: the formula in Proposition 6 apply directly with the simple adjustment that all the estimated sufficient statistics ($D_l(s)$, $D_k(s)$, $D_{IMRPK}(s)$, $DV_{IMRPK}(s)$ and $DCOV(s)$) have to be set to 0 for industries where the experiment is not run.

Of course, the formula in Proposition 6 can also easily be extended to the case where there is a non-zero measure set of firms in each industry that receive the treatment (ϑ_s) – this can be done in a similar way to what we did in the simpler case of Proposition 4.

4.2 Decreasing returns to scale

We now return to the simple framework of Section 2, but assume that firms' production function exhibits technological decreasing returns to scale (span of control): $y_{it} = e^{z_{it}}(k_{it}^\alpha l_{it}^{1-\alpha})^\nu$ where $\nu < 1$. Then results in Proposition 2 and 3 carry through with some simple modifications:

Proposition 7. *With decreasing technological returns to scale ν , Proposition 1 remains valid: at the steady-state of the economy, the c.d.f. of the joint distribution of capital wedges τ_i and productivity z_i is independent of w and Y : $F(z_i, \tau_i; \Theta_s)$.*

With decreasing technological returns to scale ν , extending the treatment to all firms in the economy imply a change in aggregate output of:

$$\Delta \ln(Y) = \frac{(1 + \epsilon)(1 - \nu\theta)}{\theta((1 - \alpha)\nu + (1 + \epsilon)(1 - \nu))} D_l,$$

where D_l is the sufficient statistics defined in Proposition 2. Similarly, the same counterfactual leads to a change in aggregate TFP of:

$$\Delta \ln(TFP) = \frac{1 - (1 - \alpha)\nu\theta}{\theta} D_l - \alpha\nu D_k,$$

where again D_k is the sufficient statistics defined in Proposition 2.

In addition, if we make the parametric assumptions 10, then the change in aggregate output following a generalization of the treatment to all firms in the economy is given by:

$$\Delta \ln Y = -\frac{\alpha\nu(1 + \epsilon)}{(1 - \alpha)\nu + (1 + \epsilon)(1 - \nu)} \left(D_{IMRPK} + \frac{1}{2} \frac{\alpha\nu\theta}{1 - \nu\theta} DV_{IMRPK} + DCOV \right),$$

and the change in aggregate TFP:

$$\Delta \ln(TFP) = -\frac{1}{2}\alpha\nu\frac{1 - (1 - \alpha)\nu\theta}{1 - \nu\theta}DV_{IMRPK}$$

where the sufficient statistics D_{IMRPK} , DV_{IMRPK} and $DCOV$ are defined in Proposition 3.

Proof. See Appendix B.6. □

Proposition 7 makes clear that our approach also applies to models of perfect competition and decreasing returns to scale (DRS) such as [Hopenhayn \(2014\)](#) or [Midrigan and Xu \(2014\)](#).

4.3 CES production function

TO BE COMPLETED

5 Empirical application: TO BE COMPLETED

6 Conclusion

This paper develops a simple sufficient statistics framework to perform aggregate counterfactuals using well-identified evidence of capital distortions. The methodology proceeds in two steps: (1) the empirical experiment is used to recover causal estimates of sufficient statistics that characterize how distortions are affected by an empirical “treatment” (2) using these estimated statistics within a general equilibrium model to infer changes in aggregate outcomes that would result from extending the treatment to all firms in the economy. We provide exact formulas, as well as parametric approximations to account for potential skewness in the distribution of productivity and capital wedges.

The methodology can only be applied when the distribution of capital wedges is independent of general equilibrium quantities. We show this is the case in a generic class of macro-finance models where (1) intermediate inputs are combined with (nests of) CES aggregators (2) production takes place according to a Cobb-Douglas technology with labor and capital (3) capital adjustment costs, financing frictions and taxes satisfy some (form of) homogeneity property (which is satisfied, e.g., by neoclassical adjustment costs, collateral constrained, linear equity issuance costs, etc.).

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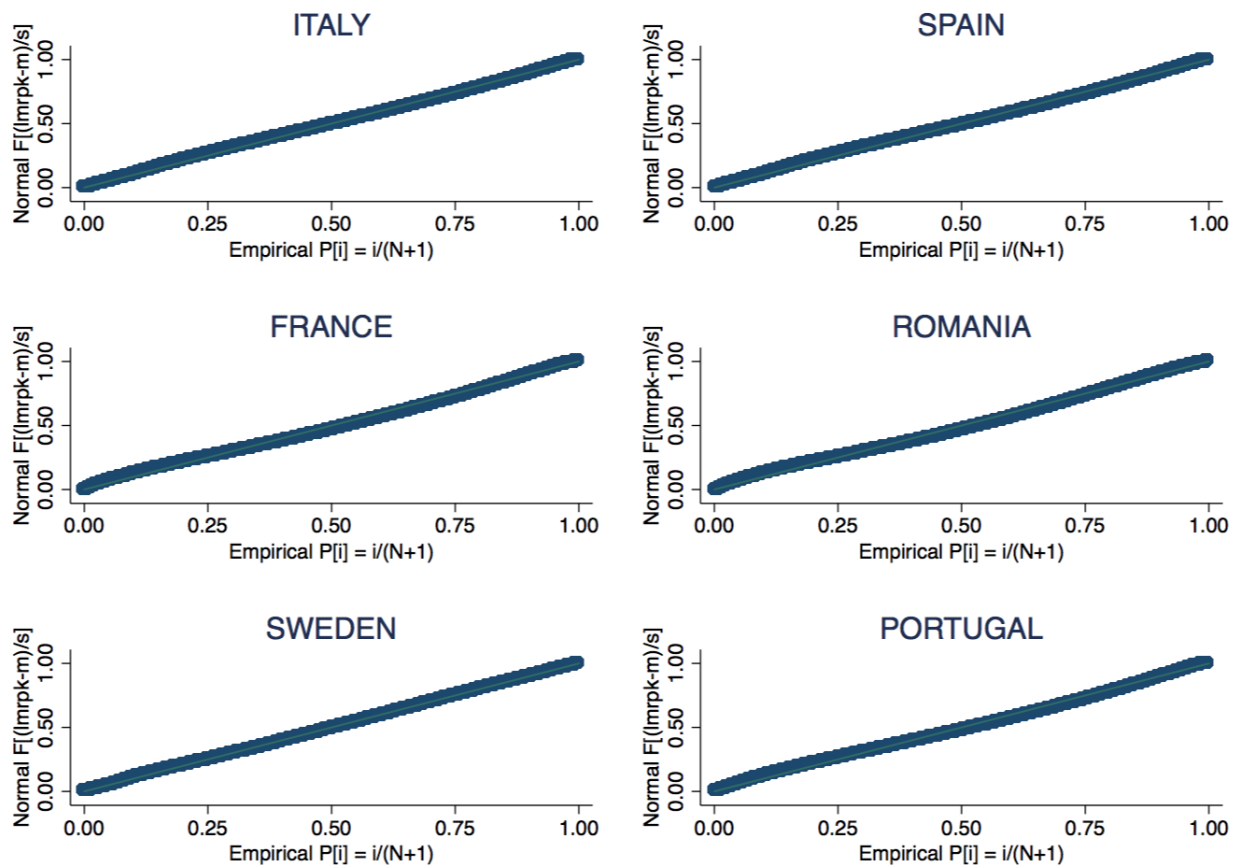
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A Figures and Tables

Figure 1: Normal probability plot of log-MRPK for firms in Amadeus



Source: BvD AMADEUS Financials, 2014. Note: This figure shows normal probability plots for 6 OECD countries (France, Spain, Italy, Portugal, Romania and Sweden) for the distribution of log-MRPK. Log-MRPK is computed as the ratio of value added (operating revenue minus materials) and total fixed assets.

B Proofs

B.1 Proof of Proposition 1

Remember that equity issuance / distributions are given by:

$$e_{it} = \frac{\alpha}{\alpha + (1 - \alpha)\phi} \left(\frac{(1 - \alpha)\phi}{\alpha + (1 - \alpha)\phi} \right)^{\frac{1 - \alpha}{\alpha}\phi} S_t^{1 - \phi} e^{\frac{\phi}{\alpha} z_{it}} k_{it}^\phi - (k_{t+1} - (1 - \delta)k_t) - \Gamma(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) \\ + \left(\frac{b_{it+1}}{1 + r(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t)} - b_{it} \right) - \mathcal{T}(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t),$$

where $S_t = \frac{Y_t}{\frac{(1 - \alpha)\phi}{w_t^{\alpha(1 - \phi)}}$. By combining the different assumptions in Proposition 1, we get that:

$$e_{it} = S_t \left(\frac{\alpha}{\alpha + (1 - \alpha)\phi} \left(\frac{(1 - \alpha)\phi}{\alpha + (1 - \alpha)\phi} \right)^{\frac{1 - \alpha}{\alpha}\phi} e^{\frac{\phi}{\alpha} z_{it}} \left(\frac{k_{it}}{S_t} \right)^\phi - \left(\frac{k_{t+1}}{S_t} - (1 - \delta) \frac{k_t}{S_t} \right) \right. \\ \left. - \Gamma\left(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1\right) + \left(\frac{b_{it+1}}{1 + r(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1)} - \frac{b_{it}}{S_t} \right) - \mathcal{T}\left(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1\right) \right)$$

Therefore, $e(z_{it}, \mathbf{x}_{it}; \Theta, w_t, Y_t) = S_t e(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1)$. Since the equity issuance cost $\mathcal{C}()$ also satisfies property 3, the flow variable in the Bellman equation 2 can be rewritten as:

$$e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) = S \times \left(e(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1) - \mathcal{C}(z_{it}, \frac{\mathbf{x}_{it}}{S_t}; \Theta, 1, 1) \right)$$

We now consider the steady-state of this economy: $w_t = w_{t+1} = w$ and $Y_t = Y_{t+1} = Y$. The Bellman equation 2 becomes:

$$V(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \frac{\mathbb{E}_{z'}[V(z', k', b'; \Theta, w, Y)|z]}{1 + r_f} \\ \mathbf{M}(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases}$$

Let \mathcal{B} be the Bellman operator:

$$\mathcal{B}f(z, k, b; \Theta, w, Y) = \begin{cases} \max_{k', b'} e(z, \mathbf{x}; \Theta, w, Y) - \mathcal{C}(z, \mathbf{x}; \Theta, w, Y) + \frac{\mathbb{E}_{z'}[f(z', k', b'; \Theta, w, Y)|z]}{1 + r_f} \\ \mathbf{M}(z, \mathbf{x}; \Theta, w, Y) \leq 0 \end{cases}$$

Consider the set of functions \mathcal{F} such that for all $(z, k, b; \Theta, w, Y)$, $f(z, k, b; \Theta, w, Y) =$

$S \times f(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)$. If $f \in \mathcal{F}$, then $\mathcal{B}f \in \mathcal{F}$:

$$\begin{aligned}
\mathcal{B}f(z, k, b; \Theta, w, Y) &= \begin{cases} \max_{k', b'} S \times \left(e(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) - \mathcal{C}(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)|z]}{1+r_f} \right) \\ M(z, \frac{\mathbf{x}}{S}; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \begin{cases} \max_{k', b'} e(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) - \mathcal{C}(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \Theta, 1, 1)|z]}{1+r_f} \\ M(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \begin{cases} \max_{k', b'} e(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) - \mathcal{C}(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', k', b'; \Theta, 1, 1)|z]}{1+r_f} \\ M(z, \frac{k}{S}, k', \frac{b}{S}, b'; \Theta, 1, 1) \leq 0 \end{cases} \\
&= S \times \mathcal{B}f(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1)
\end{aligned}$$

Since the contraction mapping theorem applies and \mathcal{F} is a compact space, this implies that the value function V also belongs to \mathcal{F} :

$$V(z, k, b; \Theta, w, Y) = S \times V(z, \frac{k}{S}, \frac{b}{S}; \Theta, 1, 1).$$

The previous equations also show that, in an economy with scale (w, Y) , if (k', b') are the optimal policies for a firm with state variable (z, k, b) , then $(\frac{k'}{S}, \frac{b'}{S})$ are the optimal policies for a firm with state variables $(z, \frac{k}{S}, \frac{b}{S})$ and in the economy with scale $(w = 1, Y = 1)$. As a result, the ergodic distribution of $\frac{k}{S}$ in the economy (w, Y) is equal to the ergodic distribution of k in the economy $(1, 1)$.

Remember that, by definition in the steady-state, capital wedges are equal to:

$$(1 + \tau_{it}) = \frac{\alpha \theta}{r_f + \delta} \frac{p_{it} y_{it}}{k_{it}} = \frac{\alpha \phi}{(\alpha + (1 - \alpha) \phi)(r_f + \delta)} e^{\frac{\phi}{\alpha} z_{it}} \left(\frac{k_{it}}{S} \right)^\phi$$

Since the ergodic distribution of $(\frac{k}{S})$ in the economy (w, Y) is the same as the ergodic distribution of k in the economy $(1, 1)$ and since the distribution of z is independent of (w, Y) , this implies that, in the steady state, the distribution of wedges τ_{it} does not depend on (w, Y) and can be written $G(\tau_i; \Theta)$.

B.2 Proof of Proposition 2

Optimal labor demand as a function of firm-level capital wedge is:

$$l_{it} = \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{1-\alpha\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_{it}} \left(\frac{\alpha\theta}{(r_f + \delta)(1 + \tau_{it})} \right)^{\frac{\alpha\theta}{1-\theta}}$$

Therefore, in the economy where there is only a zero-measure set of firms receiving the treatment, average employment for treated firms ($T=1$ and $\Theta = \Theta_1$) is given by:

$$\begin{aligned} \mathbb{E}[l_{it}|T = 1, \Theta = \Theta_1; (\bar{w}, Y_0)] &= \mathbb{E}[l_{it}|\Theta = \Theta_1; (\bar{w}, Y_0)] \\ &= \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^{\frac{1-\alpha\theta}{1-\theta}} \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha\theta}{1-\theta}} Y_0 \int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_1), \end{aligned}$$

where we use the fact that treatment is randomized and the ergodic distribution of (z, τ) for the subset of firms operating with $\Theta = \Theta_1$ is independent of (w, Y) and (\bar{w}, Y_0) corresponds to the equilibrium wage and aggregate output in the economy where only a zero-measure set of firms receive the treatment.

Similarly, for firms in the control group:

$$\begin{aligned} \mathbb{E}[l_{it}|T = 0, \Theta = \Theta_0; (\bar{w}, Y_0)] &= \mathbb{E}[l_{it}|\Theta = \Theta_0; (\bar{w}, Y_0)] \\ &= \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^{\frac{1-\alpha\theta}{1-\theta}} \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha\theta}{1-\theta}} Y_0 \int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_0), \end{aligned}$$

Therefore, the estimator D_l introduced in Proposition 2 corresponds to:

$$\begin{aligned} D_l &= \ln(\mathbb{E}[l_{it}|T = 1, \Theta = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[l_{it}|T = 0, \Theta = \Theta_0; (\bar{w}, Y_0)]) \\ &= \ln(\mathbb{E}[l_{it}|\Theta = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[l_{it}|\Theta = \Theta_0; (\bar{w}, Y_0)]) \text{ as treatment is randomized} \\ &= \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_0) \right) \end{aligned}$$

Remember that $\Delta \ln(Y) = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right]$

Therefore: $\Delta \ln(Y) = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} D_l$

Similarly, optimal capital demand implies that:

$$k_{it} = \left(\frac{(1-\alpha)\theta}{w} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_{it}} \left(\frac{\alpha\theta}{(r_f + \delta)(1 + \tau_{it})} \right)^{\frac{1-(1-\alpha)\theta}{1-\theta}}$$

Therefore, in the economy where there is only a zero-measure set of firms receiving the treatment, average capital for treated firms ($T=1$ and $\Theta = \Theta_1$) is given by:

$$\begin{aligned} \mathbb{E}[k_{it}|T = 1, \Theta = \Theta_1; (\bar{w}, Y_0)] &= \mathbb{E}[k_{it}|\Theta = \Theta_1; (\bar{w}, Y_0)] \\ &= \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{1-(1-\alpha)\theta}{1-\theta}} Y_0 \int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \end{aligned}$$

where we again use the fact that treatment is randomized and the ergodic distribution of (z, τ) for the subset of firms operating with $\Theta = \Theta_1$ is independent of (w, Y) and (\bar{w}, Y_0) corresponds to the equilibrium wage and aggregate output in the economy where only a zero-measure set of firms receive the treatment.

Similarly, for firms in the control group:

$$\mathbb{E}[k_{it}|T = 0, \Theta = \Theta_0; (\bar{w}, Y_0)] = \mathbb{E}[k_{it}|\Theta = \Theta_0; (\bar{w}, Y_0)] = \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{r} \right)^{\frac{1-(1-\alpha)\theta}{1-\theta}} Y_0 \int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0)$$

Therefore, the estimator D_k introduced in Proposition 2 corresponds to:

$$\begin{aligned} D_k &= \ln(\mathbb{E}[l_{it}|T = 1, \Theta = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[k_{it}|T = 0, \Theta = \Theta_0; (\bar{w}, Y_0)]) \\ &= \ln(\mathbb{E}[k_{it}|\Theta = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[k_{it}|\Theta = \Theta_0; (\bar{w}, Y_0)]) \quad \text{as treatment is randomized} \\ &= \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \end{aligned}$$

Remember that:

$$\begin{aligned} \Delta \ln(TFP) &= \frac{1 - (1-\alpha)\theta}{\theta} \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right] \\ &\quad - \alpha \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right] \end{aligned}$$

Therefore $\Delta \ln(TFP) = \frac{1-(1-\alpha)\theta}{\theta} D_l - \alpha D_k$

B.3 Proof of Proposition 3

With the parametric assumptions in Assumption 10, aggregate outcomes are easily expressed as a function of changes in the joint distribution of wedges and productivity, which are now summarized by the average capital wedge μ_τ , the variance of capital wedges σ_τ^2 and the covariance of productivity and wedges $\sigma_{\tau z}$. The average productivity μ_z is by assumption independent of the experiment.

The effect of the experiment on these moments of the joint distribution of wedges and productivity can be estimated through simple transforms of observable quantities. Thanks to the Cobb-Douglas assumption for production, $1 + \tau_{it} = \frac{\alpha\theta}{r} \frac{p_{it}y_{it}}{k_{it}}$ and as a result:

$$\begin{aligned}
D_{IMRPK} &= \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)\right] - \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)\right] \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)] - \mathbb{E}[\ln(1 + \tau_{it}) \mid T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)] \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_1; (\bar{w}, Y_0)] - \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_0; (\bar{w}, Y_0)] \text{ as treatment is random} \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_1] - \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_0] \text{ thanks to Proposition 1} \\
&= \mu_\tau(\Theta = \Theta_1) - \mu_\tau(\Theta = \Theta_0) \\
&= \Delta\mu_\tau
\end{aligned}$$

The same proof applies exactly to the difference in the variance of IMRPK for treated and control firms:

$$\begin{aligned}
DV_{IMRPK} &= \mathbb{V}\text{ar}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)\right] - \mathbb{V}\text{ar}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)\right] \\
&= \sigma_\tau^2(\Theta = \Theta_1) - \sigma_\tau^2(\Theta = \Theta_0) \\
&= \Delta\sigma_\tau^2
\end{aligned}$$

Finally, to estimate the effect of the treatment on the covariance of wedges and productivity, we start by writing firm-level output:

$$\ln(p_{it}y_{it}) = \ln\left(\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{r}\right)^{\frac{\alpha\theta}{1-\theta}}\right) + \frac{\theta}{1-\theta} \ln(z_{it}) - \frac{\alpha\theta}{1-\theta} \ln(1 + \tau_{it})$$

As a result, in the economy where only a 0-measure set of firms receive the treatment

(\bar{w}, Y_0) , we have, for firms receiving the treatment ($T_{it} = 1$ and $\Theta = \Theta_1$):

$$\begin{aligned}
& \text{Cov} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 1, \Theta = \Theta_1; (\bar{w}, Y_0) \right] \\
&= \text{Cov} \left[\ln(1 + \tau_{it}), \ln(p_{it}y_{it}) | T_{it} = 1, \Theta = \Theta_1; (\bar{w}, Y_0) \right] \\
&= \text{Cov} \left[\ln(1 + \tau_{it}), \frac{\theta}{1 - \theta} \ln(z_{it}) - \frac{\alpha\theta}{1 - \theta} \ln(1 + \tau_{it}) | T_{it} = 1, \Theta = \Theta_1; (\bar{w}, Y_0) \right] \\
&= \frac{\theta}{1 - \theta} \text{Cov} [z_{it}, \ln(1 + \tau_{it}) | T_{it} = 1, \Theta = \Theta_1; (\bar{w}, Y_0)] - \frac{\alpha\theta}{1 - \theta} \text{Var} [z_{it} | T_{it} = 1, \Theta = \Theta_1; (\bar{w}, Y_0)] \\
&= \frac{\theta}{1 - \theta} \text{Cov} [z_{it}, \ln(1 + \tau_{it}) | \Theta = \Theta_1; (\bar{w}, Y_0)] - \frac{\alpha\theta}{1 - \theta} \text{Var} [z_{it} | \Theta = \Theta_1; (\bar{w}, Y_0)] \quad \text{as treatment is random} \\
&= \frac{\theta}{1 - \theta} \text{Cov} [z_{it}, \ln(1 + \tau_{it}) | \Theta = \Theta_1] - \frac{\alpha\theta}{1 - \theta} \text{Var} [z_{it} | \Theta = \Theta_1] \quad \text{thanks to Proposition 1} \\
&= \frac{\theta}{1 - \theta} \sigma_{\tau z}(\Theta = \Theta_1) - \frac{\alpha\theta}{1 - \theta} \sigma_{\tau}^2(\Theta = \Theta_1)
\end{aligned}$$

Similarly for firms that do not receive the treatment:

$$\text{Cov} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 0, \Theta = \Theta_0; (\bar{w}, Y_0) \right] = \frac{\theta}{1 - \theta} \sigma_{\tau z}(\Theta = \Theta_0) - \frac{\alpha\theta}{1 - \theta} \sigma_{\tau}^2(\Theta = \Theta_0)$$

As a result, we can compute the sufficient statistics DCOV as:

$$\begin{aligned}
DCOV &= \text{Cov} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0) \right] - \text{Cov} \left[\ln \left(\frac{p_{it}y_{it}}{k_{it}} \right), \ln(p_{it}y_{it}) | T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0) \right] \\
&= \frac{\theta}{1 - \theta} \Delta \sigma_{\tau z} - \frac{\alpha\theta}{1 - \theta} \Delta \sigma_{\tau}^2
\end{aligned}$$

We know from Equations 12 that changes in aggregate output can be written as:

$$\Delta \ln Y = \frac{\alpha(1 + \epsilon)}{1 - \alpha} \left[-\Delta \mu_{\tau} + \frac{1}{2} \frac{\theta}{1 - \theta} (\alpha \Delta \sigma_{\tau}^2 - 2 \Delta \sigma_{\tau z}) \right]$$

It is direct to combine the previous expressions to obtain:

$$\Delta \ln Y = -(1 + \epsilon) \frac{\alpha}{1 - \alpha} \left(D_{IMRPK} + \frac{1}{2} \frac{\alpha\theta}{1 - \theta} DV_{IMRPK} + DCOV \right)$$

Similarly, Equations 12 shows that changes in aggregate TFP are written as:

$$\Delta \ln TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1 - \theta} \right) \Delta \sigma_{\tau}^2,$$

which can be rewritten as:

$$\Delta \ln TFP = -\frac{\alpha}{2} \left(1 + \frac{\alpha\theta}{1-\theta} \right) DV_{IMRPK}$$

B.4 Proof of Proposition 4

In counterfactual 1, changes in aggregate output is given by the same formula as in the small experiment case:

$$\Delta \ln(Y) = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \left[\ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_0) \right) \right]$$

Let $(w_\vartheta, Y_\vartheta)$ be the equilibrium wage and output in the actual economy where the experiment is realized and a fraction ϑ of firms receive the treatment. In this economy, the estimated statistics D_l corresponds to:

$$\begin{aligned} D_l &= \ln(\mathbb{E}[l_{it}|T=1, \Theta=\Theta_1; (w_\vartheta, Y_\vartheta)]) - \ln(\mathbb{E}[l_{it}|T=0, \Theta=\Theta_0; (w_\vartheta, Y_\vartheta)]) \\ &= \ln(\mathbb{E}[l_{it}|\Theta=\Theta_1; (w_\vartheta, Y_\vartheta)]) - \ln(\mathbb{E}[l_{it}|\Theta=\Theta_0; (w_\vartheta, Y_\vartheta)]) \text{ as treatment is randomized} \\ &= \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_1) \right) - \ln \left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_{it}}}{(1+\tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}, \Theta_0) \right) \end{aligned}$$

In other words, the estimated statistics D_l does not depend on the current macroeconomic conditions, and therefore can be again applied directly to compute the counterfactual change in aggregate output. The same argument applies to the estimation of changes in aggregate TFP using the estimated sufficient statistics D_k . By the same token, under the parametric assumptions 10, the formula in Proposition 3 would continue to apply to this case.

Consider now counterfactual 2, in which we are trying to compute the change in aggregate output and TFP of going from the actual economy where a fraction $\vartheta > 0$ of firms receive the treatment (characterized by $(w_\vartheta, Y_\vartheta)$) to an economy where all firms receive the treatment $((w_1, Y_1))$.

It is direct to see that in the *actual* economy, aggregate output is given by:

$$\frac{Y_\vartheta}{\bar{L}} = \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^\epsilon \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha(1+\epsilon)}{1-\alpha}} \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} (\vartheta dF(z_{it}, \tau_{it}; \Theta_1) + (1-\vartheta) dF(z_{it}, \tau_{it}; \Theta_0)) \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}}$$

In the counterfactual economy where all firms are treated, aggregate output is instead given by:

$$\frac{Y_1}{\bar{L}} = \left(\frac{(1-\alpha)\theta}{\bar{w}} \right)^\epsilon \left(\frac{\alpha\theta}{r_f + \delta} \right)^{\frac{\alpha(1+\epsilon)}{1-\alpha}} \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta_1) \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}}$$

Therefore, change in aggregate output from the actual economy to the counterfactual economy is given by:

$$\begin{aligned} \Delta \ln(Y) &= \ln(Y_1) - \ln(Y_\vartheta) \\ &= \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \left[\ln \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta_1) \right) \right. \\ &\quad \left. - \ln \left(\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} (\vartheta dF(z_{it}, \tau_{it}; \Theta_1) + (1-\vartheta)dF(z_{it}, \tau_{it}; \Theta_0)) \right) \right] \end{aligned}$$

With the parametric assumptions in Assumption 10, we know that:

$$\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z_{it}, \tau_{it}; \Theta_1) = e^{\frac{\theta}{1-\theta}(\mu_z - \alpha\mu_\tau(\Theta_1)) + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2(\sigma_z^2 + \alpha^2\sigma_\tau^2(\Theta_1) - 2\alpha\sigma_{\tau z}(\Theta_1))},$$

and:

$$\begin{aligned} &\int_i \left(\frac{e^{z_{it}}}{(1+\tau_{it})^\alpha} \right)^{\frac{\theta}{1-\theta}} (\vartheta dF(z_{it}, \tau_{it}; \Theta_1) + (1-\vartheta)dF(z_{it}, \tau_{it}; \Theta_0)) \\ &= \vartheta e^{\frac{\theta}{1-\theta}(\mu_z - \alpha\mu_\tau(\Theta_1)) + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2(\sigma_z^2 + \alpha^2\sigma_\tau^2(\Theta_1) - 2\alpha\sigma_{\tau z}(\Theta_1))} + (1-\vartheta) e^{\frac{\theta}{1-\theta}(\mu_z - \alpha\mu_\tau(\Theta_0)) + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2(\sigma_z^2 + \alpha^2\sigma_\tau^2(\Theta_0) - 2\alpha\sigma_{\tau z}(\Theta_0))} \end{aligned}$$

so that the previous expression can be transformed into

$$\Delta \ln(Y) = -\frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \ln \left(\vartheta + (1-\vartheta) e^{\frac{\alpha\theta}{1-\theta}\Delta\mu_\tau + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2(-\alpha^2\Delta\sigma_\tau^2 + 2\alpha\Delta\sigma_{\tau z})} \right)$$

As in Proposition 3, $\Delta\mu_\tau$, $\Delta\sigma_\tau^2$ and $\Delta\sigma_{\tau z}$ can be estimated using the sufficient statistics introduced in Proposition 3. For instance, D_{IMRPK} is, in the context of this large experi-

ment:

$$\begin{aligned}
D_{IMRPK} &= \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 1, \Theta_{it} = \Theta_1; (w_{\vartheta}, Y_{\vartheta})\right] - \mathbb{E}\left[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) \mid T_{it} = 0, \Theta_{it} = \Theta_0; (w_{\vartheta}, Y_{\vartheta})\right] \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid T_{it} = 1, \Theta_{it} = \Theta_1; (w_{\vartheta}, Y_{\vartheta})] - \mathbb{E}[\ln(1 + \tau_{it}) \mid T_{it} = 0, \Theta_{it} = \Theta_0; (w_{\vartheta}, Y_{\vartheta})] \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_1; (w_{\vartheta}, Y_{\vartheta})] - \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_0; (w_{\vartheta}, Y_{\vartheta})] \quad \text{as treatment is random} \\
&= \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_1] - \mathbb{E}[\ln(1 + \tau_{it}) \mid \Theta_{it} = \Theta_0] \quad \text{thanks to Proposition 1} \\
&= \mu_{\tau}(\Theta = \Theta_1) - \mu_{\tau}(\Theta = \Theta_0) \\
&= \Delta\mu_{\tau}
\end{aligned}$$

Through the same token: $DV_{IMRPK} = \Delta\sigma_{\tau}^2$ and $DCOV = \frac{\theta}{1-\theta}\Delta\sigma_{\tau z} - \frac{\alpha\theta}{1-\theta}\Delta\sigma_{\tau}^2$.

As a result, we see that the change in aggregate output resulting from generalizing the treatment to all firms in the economy, starting from the *actual* economy can be estimated as:

$$\Delta \ln(Y) = -\frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \ln\left(\vartheta + (1-\vartheta)e^{\frac{\alpha\theta}{1-\theta}(D_{IMRPK} + \frac{1}{2}\frac{\alpha\theta}{1-\theta}DV_{IMRPK} + DCOV)}\right)$$

We now turn to the TFP counterfactual. We have that:

$$\begin{aligned}
\Delta \ln(TFP) &= \frac{1 - (1 - \alpha)\theta}{\theta} \left[\ln\left(\int_i \frac{e^{\frac{\theta}{1-\theta}z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1)\right) \right. \\
&\quad \left. - \ln\left(\int_i \frac{e^{\frac{\theta}{1-\theta}z_{it}}}{(1 + \tau_{it})^{\frac{\alpha\theta}{1-\theta}}} (\vartheta dF(z_{it}, \tau_{it}; \Theta_1) + (1 - \vartheta)dF(z_{it}, \tau_{it}; \Theta_0))\right)\right] \\
&\quad - \alpha \left[\ln\left(\int_i \frac{e^{\frac{\theta}{1-\theta}z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z_{it}, \tau_{it}; \Theta_1)\right) \right. \\
&\quad \left. - \ln\left(\int_i \frac{e^{\frac{\theta}{1-\theta}z_{it}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha)\theta}{1-\theta}}} (\vartheta dF(z_{it}, \tau_{it}; \Theta_1) + (1 - \vartheta)dF(z_{it}, \tau_{it}; \Theta_0))\right)\right]
\end{aligned}$$

With the parametric assumptions 10, this last expression becomes:

$$\begin{aligned}
\Delta \ln(TFP) &= -\frac{1-(1-\alpha)\theta}{\theta} \ln \left(\vartheta + (1-\vartheta) e^{\frac{\alpha\theta}{1-\theta} \Delta\mu_\tau + \frac{1}{2} \left(\frac{\theta}{1-\theta}\right)^2 (-\alpha^2 \Delta\sigma_\tau^2 + 2\alpha \Delta\sigma_{\tau z})} \right) \\
&\quad + \alpha \ln \left(\vartheta + (1-\vartheta) e^{\frac{1-(1-\alpha)\theta}{1-\theta} \Delta\mu_\tau + \frac{1}{2} \left(\frac{1}{1-\theta}\right)^2 (-(1-(1-\alpha)\theta)^2 \Delta\sigma_\tau^2 + 2\theta(1-(1-\alpha)\theta) \Delta\sigma_{\tau z})} \right) \\
&= -\ln \left(\frac{\left(\vartheta + (1-\vartheta) e^{\frac{\alpha\theta}{1-\theta} \Delta\mu_\tau + \frac{1}{2} \left(\frac{\theta}{1-\theta}\right)^2 (-\alpha^2 \Delta\sigma_\tau^2 + 2\alpha \Delta\sigma_{\tau z})} \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\vartheta + (1-\vartheta) e^{\frac{1-(1-\alpha)\theta}{1-\theta} \Delta\mu_\tau + \frac{1}{2} \left(\frac{1}{1-\theta}\right)^2 (-(1-(1-\alpha)\theta)^2 \Delta\sigma_\tau^2 + 2\theta(1-(1-\alpha)\theta) \Delta\sigma_{\tau z})} \right)^\alpha} \right)
\end{aligned}$$

In terms of sufficient statistics, this is equivalent to:

$$\Delta \ln(TFP) = -\ln \left(\frac{\left(\vartheta + (1-\vartheta) e^{\frac{\alpha\theta}{1-\theta} (D_{IMRPK} + \frac{1}{2} \frac{\alpha\theta}{1-\theta} DV_{IMRPK} + DCOV)} \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\vartheta + (1-\vartheta) e^{\frac{1-(1-\alpha)\theta}{1-\theta} (D_{IMRPK} + DCOV + \frac{\alpha\theta - (1-\theta)}{2(1-\theta)} DV_{IMRPK})} \right)^\alpha} \right)$$

B.5 Proof for Proposition 6

With monopolistic competition, industry prices of p_s , industry output of Y_s and a price elasticity of demand $-\frac{1}{1-\theta_s}$, for a firm i operating in industry s with a stock of capital k_{it} , operating profits after optimization labor demand are given by:

$$p_{it}y_{it} - wl_{it} = (1 - (1 - \alpha_s)\theta_s) \left(\frac{(1 - \alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-(1-\alpha_s)\theta_s}} p_s^{\frac{1}{1-(1-\alpha_s)\theta_s}} Y_s^{\frac{1-\theta_s}{1-(1-\alpha_s)\theta_s}} e^{z_{it} \frac{\theta_s}{1-(1-\alpha_s)\theta_s}} k_{it}^{\frac{\alpha_s\theta_s}{1-(1-\alpha_s)\theta_s}}$$

Since the final good is produced from industry output with a Cobb-Douglas aggregator, industry shares in output are fixed:

$$p_s Y_s = \phi_s Y, \quad \text{for all } s \in [1, S]$$

As a result, firm-level operating profits can be rewritten as:

$$\begin{aligned} p_{it}y_{it} - wl_{it} &= (1 - (1 - \alpha_s)\theta_s) \left(\frac{(1 - \alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-(1-\alpha_s)\theta_s}} \phi_s^{\frac{1}{1-(1-\alpha_s)\theta_s}} Y^{\frac{1}{1-(1-\alpha_s)\theta_s}} Y_s^{\frac{-\theta_s}{1-(1-\alpha_s)\theta_s}} e^{z_{it} \frac{\theta_s}{1-(1-\alpha_s)\theta_s}} k_{it}^{\frac{\alpha_s\theta_s}{1-(1-\alpha_s)\theta_s}} \\ &= (1 - (1 - \alpha_s)\theta_s) \left(\frac{(1 - \alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-(1-\alpha_s)\theta_s}} \phi_s^{\frac{1}{1-(1-\alpha_s)\theta_s}} S_{st} e^{z_{it} \frac{\theta_s}{1-(1-\alpha_s)\theta_s}} \left(\frac{k_{it}}{S_{st}} \right)^{\frac{\alpha_s\theta_s}{1-(1-\alpha_s)\theta_s}}, \end{aligned}$$

where $S_{st} = \frac{Y^{\frac{1}{1-\theta_s}}}{w^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} Y_s^{\frac{\theta_s}{1-\theta_s}}}$.

It follows directly from the proof of Proposition 1 that in this economy, and under the assumptions of Proposition 1, the ergodic joint distribution of capital wedges and productivity is independent of (w, Y_s, Y) and depend only on the parameters Θ_s governing this industry, as well the structural parameters $(\alpha_s, \phi_s, \theta_s)$. Let $F_s(z, \tau; \Theta_s)$ denote this distribution for industry s .

Profit maximization for firm i in industry s as a function of a capital wedge τ_{is} leads to:

$$\begin{cases} k_{it} = \left(\frac{(1 - \alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} p_s^{\frac{1}{1-\theta_s}} Y_s e^{z_{it} \frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r(1 + \tau_{it})} \right)^{\frac{1-(1-\alpha_s)\theta_s}{1-\theta_s}} \\ l_{it} = \left(\frac{(1 - \alpha_s)\theta_s}{w} \right)^{\frac{1-\alpha_s\theta_s}{1-\theta_s}} p_s^{\frac{1}{1-\theta_s}} Y_s e^{z_{it} \frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r(1 + \tau_{it})} \right)^{\frac{\alpha_s\theta_s}{1-\theta_s}} \end{cases}$$

Firm i production at the optimum is given by:

$$y_{it} = e^{z_{it}} k_{it}^{\alpha_s} l_{it}^{1-\alpha_s} = \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{1-\alpha_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r(1+\tau_{it})} \right)^{\frac{\alpha_s}{1-\theta_s}} e^{\frac{z_{it}}{(1-\theta_s)}} p_s^{\frac{1}{1-\theta_s}} Y_s$$

Total production in industry s can thus be written as:

$$(Y_s)^{\theta_s} = \int_{i \in M_s} y_{it}^{\theta_s} \Rightarrow 1 = \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r} \right)^{\frac{\alpha_s\theta_s}{1-\theta_s}} p_s^{\frac{\theta_s}{1-\theta_s}} I_s,$$

where $I_s = n_s \int_{(z,\tau)} \left(\frac{z_i}{(1+\tau_i)\alpha_s} \right)^{\frac{\theta_s}{1-\theta_s}} dF_s(z, \tau; \Theta_s)$, and n_s is the fraction of firms in industry s . This implies that:

$$\ln(p_s) = -(1-\alpha_s) \ln \left(\frac{(1-\alpha_s)\theta_s}{w} \right) - \alpha_s \ln \left(\frac{\alpha_s\theta_s}{r} \right) - \frac{1-\theta_s}{\theta_s} \ln(I_s) \quad (17)$$

We then write down aggregate production, using the fact that $p_s Y_s = \phi_s Y$:

$$\ln(Y) = \sum_s \phi_s \ln(Y_s) \Rightarrow 0 = \Upsilon - \sum_s \phi_s \ln(p_s),$$

where $\Upsilon = \sum_{s=1}^S \phi_s \ln(\phi_s)$. Using the expression for p_s yields the equilibrium wage:

$$w = \prod_{s=1}^S \left[\left((1-\alpha_s) \right)^{\frac{(1-\alpha_s)\phi_s}{\sum_{l=1}^S (1-\alpha_l)\phi_l}} \left(\frac{\alpha_s}{r} \right)^{\frac{\alpha_s\phi_s}{\sum_{l=1}^S (1-\alpha_l)\phi_l}} (\phi_s\theta_s)^{\frac{\phi_s}{\sum_{l=1}^S (1-\alpha_l)\phi_l}} I_s^{\frac{\phi_s(1-\theta_s)}{\theta_s \sum_{l=1}^S (1-\alpha_l)\phi_l}} \right]$$

We then write down aggregate labor demand, summing-up firm-level labor demand and using the fact that $Y_s = \phi_s \frac{Y}{p_s}$:

$$\begin{aligned} L^d &= Y \sum_{s=1}^S \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{1-\alpha_s\theta_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r} \right)^{\frac{\alpha_s\theta_s}{1-\theta_s}} \phi_s p_s^{\frac{\theta_s}{1-\theta_s}} I_s \\ &= Y \sum_{s=1}^S \phi_s \underbrace{\left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s\theta_s}{r} \right)^{\frac{\alpha_s\theta_s}{1-\theta_s}} p_s^{\frac{\theta_s}{1-\theta_s}} I_s}_{=1 \text{ (from Equation 17)}} \\ &= \frac{Y}{w} \sum_{s=1}^S \phi_s (1-\alpha_s)\theta_s \end{aligned}$$

Note that industry-level employment is given by:

$$L_s = Y \frac{(1 - \alpha_s) \phi_s \theta_s}{w} = L \left(\frac{(1 - \alpha_s) \phi_s \theta_s}{\sum_{l=1}^S (1 - \alpha_l) \phi_l \theta_l} \right)$$

Labor market equilibrium implies that $L^d = \bar{L} \left(\frac{w}{\bar{w}} \right)^\epsilon$, which, using the equilibrium expression for the wage derived above, leads to:

$$\frac{Y}{\bar{L}} = \left(\frac{1}{\sum_{s=1}^S \phi_s \theta_s \left(\frac{1 - \alpha_s}{\bar{w}} \right)} \right) \prod_{s=1}^S \left[\left(\frac{1 - \alpha_s}{\bar{w}} \right)^{\frac{(1 - \alpha_s) \phi_s}{\sum_{l=1}^S (1 - \alpha_l) \phi_l}} \left(\frac{\alpha_s}{r} \right)^{\frac{\alpha_s \phi_s}{\sum_{l=1}^S (1 - \alpha_l) \phi_l}} (\phi_s \theta_s)^{\frac{\phi_s}{\sum_{l=1}^S (1 - \alpha_l) \phi_l}} I_s^{\frac{\phi_s (1 - \theta_s)}{\sum_{l=1}^S (1 - \alpha_l) \phi_l}} \right]^{1 + \epsilon}$$

We can also aggregate firm-level capital demand to obtain the total capital stock in the economy, using again the fact that the share of industry s output in total output is ϕ_s :

$$K = Y \sum_{s=1}^S \left(\frac{(1 - \alpha_s) \theta_s}{w} \right)^{\frac{(1 - \alpha_s) \theta_s}{1 - \theta_s}} \left(\frac{\alpha_s \theta_s}{r} \right)^{\frac{1 - (1 - \alpha_s) \theta_s}{1 - \theta_s}} \phi_s p_s^{\frac{\theta_s}{1 - \theta_s}} J_s,$$

where $J_s = n_s \int_{(z, \tau)} \frac{e^{\frac{\theta_s}{1 - \theta_s} z_i}}{(1 + \tau_i)^{\frac{1 - (1 - \alpha_s) \theta_s}{1 - \theta_s}}} dF_s(z_i, \tau_i; \Theta_s)$.

We can then use the relationship between industry s price p_s and the equilibrium wage to substitute for p_s in the previous expression and obtain:

$$K = \left(\frac{Y}{r} \right) \sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s}{I_s}$$

Note that the industry-level capital stock writes:

$$K_s = \left(\frac{Y}{r} \right) \alpha_s \phi_s \theta_s \frac{J_s}{I_s} = K \left(\frac{\alpha_s \phi_s \theta_s \frac{J_s}{I_s}}{\sum_{l=1}^S \alpha_s \phi_s \theta_s \frac{J_s}{I_s}} \right) \quad (18)$$

We also know that $L = \frac{Y}{w} \sum_{s=1}^S (1 - \alpha_s) \phi_s \theta_s$.

To compute aggregate TFP, we write that:

$$\begin{aligned}
Y &= \prod_{s=1}^S Y_s^{\phi_s} = \prod_{s=1}^S (TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s})^{\phi_s} \\
&= \underbrace{\left[\prod_{s=1}^S TFP_s^{\phi_s} \left(\frac{\alpha_s \phi_s \theta_s \frac{J_s}{I_s}}{\sum_{l=1}^S \alpha_s \phi_s \theta_s \frac{J_s}{I_s}} \right)^{\alpha_s \phi_s} \left(\frac{(1-\alpha_s) \phi_s \theta_s}{\sum_{l=1}^S (1-\alpha_l) \phi_l \theta_l} \right)^{(1-\alpha_s) \phi_s} \right]}_{=TFP} K^{\sum_{l=1}^S \alpha_l \phi_l} L^{\sum_{l=1}^S (1-\alpha_l) \phi_l}
\end{aligned}$$

Using the expression for aggregate capital and labor derived above, we can calculate the aggregate TFP term above:

$$\begin{aligned}
TFP &= \frac{Y}{K^{\sum_{s=1}^S \alpha_s \phi_s} L^{\sum_{s=1}^S (1-\alpha_s) \phi_s}} \\
&= r^{\sum_{s=1}^S \alpha_s \phi_s} w^{\sum_{s=1}^S (1-\alpha_s) \phi_s} \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s}{I_s} \right)^{-\sum_{s=1}^S \alpha_s \phi_s} \left(\sum_{s=1}^S (1-\alpha_s) \phi_s \theta_s \right)^{-\sum_{s=1}^S (1-\alpha_s) \phi_s} \\
&= (19) \left(\sum_{s=1}^S (1-\alpha_s) \phi_s \theta_s \right)^{-\sum_{s=1}^S (1-\alpha_s) \phi_s} \left[\prod_{s=1}^S \left((1-\alpha_s)^{(1-\alpha_s)} \alpha_s^{\alpha_s} \phi_s \theta_s I_s^{\frac{(1-\theta_s)}{\theta_s}} \right)^{\phi_s} \right] \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s}{I_s} \right)^{-\sum_{s=1}^S \alpha_s \phi_s}
\end{aligned}$$

We now move on to compute the aggregate counterfactuals using the experimental estimates, i.e how extending the treatment ($\Theta_s = \Theta_s^0$ becomes $\Theta_s = \Theta_s^1$) to all firms in the economy would affect aggregate output Y and aggregate TFP.

We see first that change in output $\Delta \ln(Y)$ is given by:

$$\Delta \ln Y = (1 + \epsilon) \sum_{s=1}^S \left(\frac{\phi_s (1 - \theta_s)}{\theta_s \sum_{l=1}^S (1 - \alpha_l) \phi_l} \right) \Delta \ln I_s,$$

where $\Delta \ln(I_s) = \ln(I_s(\Theta_s = \Theta_s^1)) - \ln(I_s(\Theta_s = \Theta_s^0))$.

Using firm-level labor demand in industry s , we see that:

$$\begin{aligned}\mathbb{E} [l_{it}|T_{it} = 1, s_i = s; (w, (Y_s)_s, Y)] &= \left(\frac{1 - \alpha_s}{w}\right)^{\frac{1 - \alpha_s \theta_s}{1 - \theta_s}} \left(\frac{\alpha_s}{r}\right)^{\frac{\alpha_s \theta_s}{1 - \theta_s}} (p_s \theta_s)^{\frac{1}{1 - \theta_s}} Y_s \int_{i \in M_s} \frac{e^{z_{it} \frac{\theta_s}{1 - \theta_s}}}{(1 + \tau_{it})^{\frac{\alpha_s \theta_s}{1 - \theta_s}}} dF(z, \tau; \Theta_s^1) \\ &= \left(\frac{1 - \alpha_s}{w}\right)^{\frac{1 - \alpha_s \theta_s}{1 - \theta_s}} \left(\frac{\alpha_s}{r}\right)^{\frac{\alpha_s \theta_s}{1 - \theta_s}} (p_s \theta_s)^{\frac{1}{1 - \theta_s}} Y_s \frac{I_s(\Theta_s^1)}{n_s}\end{aligned}$$

Let $D_l(s)$ be the log-treatment effect on employment in industry s . We see from the previous equation that:

$$\begin{aligned}D_l(s) &= \mathbb{E} [l_{it}|T_{it} = 1, \Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E} [l_{it}|T_{it} = 0, \Theta_{it} = \Theta_s^0, s_i = s; (w, (Y_s)_s, Y)] \\ &= \mathbb{E} [l_{it}|\Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E} [l_{it}|\Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] \\ &\quad \text{as treatment is random} \\ &= \ln(I_s(\Theta_1)) - \ln(I_s(\Theta_0)) \\ &= \Delta \ln(I_s)\end{aligned}$$

And we see that:

$$\Delta \ln Y = (1 + \epsilon) \sum_{s=1}^S \left(\frac{\phi_s(1 - \theta_s)}{\theta_s \sum_{l=1}^S (1 - \alpha_l) \phi_l} \right) D_l(s)$$

If we make the parametric assumption that, within an industry s , the joint distribution of productivity and capital wedges is log-normally distributed:

$$\begin{pmatrix} \ln(z_{it}^s) \\ \ln(1 + \tau_{it}^s) \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mu_z \\ \mu_\tau^s(\Theta) \end{pmatrix}, \begin{pmatrix} \sigma_z & \sigma_{\tau z}^s(\Theta) \\ \sigma_{\tau z}^s(\Theta) & (\sigma_\tau^s(\Theta))^2 \end{pmatrix} \right], \quad (20)$$

then we can write:

$$\ln(I_s(\Theta)) = \frac{\theta_s}{1 - \theta_s} (\mu_z - \alpha_s \mu_\tau^s(\Theta)) + \frac{1}{2} \left(\frac{\theta_s}{1 - \theta_s} \right)^2 (\alpha_s^2 (\sigma_\tau^s(\Theta))^2 + \sigma_z^2 - 2\alpha_s \sigma_{\tau z}^s(\Theta_s))$$

So that:

$$\Delta \ln(I_s) = -\alpha_s \frac{\theta_s}{1 - \theta_s} \left(\Delta \mu_\tau^s - \frac{1}{2} \frac{\theta_s}{1 - \theta_s} (\alpha_s \Delta (\sigma_\tau^s)^2 - 2\Delta \sigma_{\tau z}^s) \right)$$

Define $D_{LMRPK}(s) = \mathbb{E}[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) | T_{it} = 1, \Theta_{it}^s = \Theta_s^1, s_i = s; (\bar{w}, (Y_s)_s, Y_0)] - \mathbb{E}[\ln\left(\frac{p_{it}y_{it}}{k_{it}}\right) | T_{it} = 0, \Theta_{it}^s = \Theta_s^0, s_i = s; (\bar{w}, (Y_s)_s, Y_0)]$. D_{LMRPK} corresponds to the treatment effect on the log output to capital ratio for firms in industry s . We can define similarly $DV_{LMRPK}(s)$ and $DCOV(s)$ as the treatment effect on the variance of the log output to capital ratio for firms in industry s and the treatment effect on the covariance between log output and log output to capital ratio for firms in industry s .

It is then direct to show that:

$$\Delta \ln Y = -(1 + \epsilon) \sum_{s=1}^S \left(\frac{\alpha_s \phi_s \theta_s}{\theta_s \sum_{l=1}^S (1 - \alpha_l) \phi_l} \right) \left(D_{LMRPK}(s) + \frac{1}{2} \frac{\alpha_s \theta_s}{1 - \theta_s} DV_{LMRPK}(s) + DCOV(s) \right)$$

We now move on to compute the change in aggregate TFP following our counterfactual experiment. From equation 19 above, we see that:

$$\Delta \ln(TFP) = \sum_{s=1}^S \phi_s \frac{1 - \theta_s}{\theta_s} \Delta \ln(I_s) - \left(\sum_{s=1}^S \alpha_s \phi_s \right) \left[\ln \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_1)}{I_s(\Theta_1)} \right) - \ln \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_0)}{I_s(\Theta_0)} \right) \right]$$

As we did above, we can estimate the first term above using $D_l(s)$:

$$\sum_{s=1}^S \phi_s \frac{1 - \theta_s}{\theta_s} \Delta \ln(I_s) = \sum_{s=1}^S \phi_s \frac{1 - \theta_s}{\theta_s} D_l(s)$$

Define $\Delta_{JI}(s) = \ln \left(\frac{J_s(\Theta_1)}{I_s(\Theta_1)} \right) - \ln \left(\frac{J_s(\Theta_0)}{I_s(\Theta_0)} \right)$. By definition: $\frac{J_s(\Theta_1)}{I_s(\Theta_1)} = e^{\Delta_{JI}(s)} \times \frac{J_s(\Theta_0)}{I_s(\Theta_0)}$.

We have:

$$\begin{aligned}
\ln \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_1)}{I_s(\Theta_1)} \right) - \ln \left(\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_0)}{I_s(\Theta_0)} \right) &= \ln \left(\frac{\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_1)}{I_s(\Theta_1)}}{\sum_{l=1}^S \alpha_l \phi_l \theta_l \frac{J_l(\Theta_0)}{I_l(\Theta_0)}} \right) \\
&= \ln \left(\frac{\sum_{s=1}^S \alpha_s \phi_s \theta_s \frac{J_s(\Theta_0)}{I_s(\Theta_0)} e^{\Delta_{JI}(s)}}{\sum_{l=1}^S \alpha_l \phi_l \theta_l \frac{J_l(\Theta_0)}{I_l(\Theta_0)}} \right) \\
&= \ln \left(\sum_{s=1}^S \frac{K_s}{K} e^{\Delta_{JI}(s)} \right) \quad \text{thanks to Equation 18}
\end{aligned}$$

Using firm-level capital demand in industry s , we see that:

$$\begin{aligned}
\mathbb{E} [k_{it} | T_{it} = 1, s_i = s; (w, (Y_s)_s, Y)] &= \left(\frac{1 - \alpha_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s}{r} \right)^{\frac{1-(1-\alpha_s)\theta_s}{1-\theta_s}} (p_s \theta_s)^{\frac{1}{1-\theta_s}} Y_s \int_{i \in M_s} \frac{e^{z_{it} \frac{\theta_s}{1-\theta_s}}}{(1 + \tau_{it})^{\frac{1-(1-\alpha_s)\theta_s}{1-\theta_s}}} dI \\
&= \left(\frac{1 - \alpha_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s}{r} \right)^{\frac{1-(1-\alpha_s)\theta_s}{1-\theta_s}} (p_s \theta_s)^{\frac{1}{1-\theta_s}} Y_s J_s(\Theta_s^1)
\end{aligned}$$

Let $D_k(s)$ be the log-treatment effect on capital in industry s . We see from the previous equation that:

$$\begin{aligned}
D_k(s) &= \mathbb{E} [k_{it} | T_{it} = 1, \Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E} [k_{it} | T_{it} = 0, \Theta_{it} = \Theta_s^0, s_i = s; (w, (Y_s)_s, Y)] \\
&= \mathbb{E} [k_{it} | \Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] - \mathbb{E} [k_{it} | \Theta_{it} = \Theta_s^1, s_i = s; (w, (Y_s)_s, Y)] \\
&\quad \text{as treatment is random} \\
&= \ln(J_s(\Theta_1)) - \ln(J_s(\Theta_0)) \\
&= \Delta \ln(J_s)
\end{aligned}$$

As a result, it is direct to see that:

$$\Delta_{JI}(s) = \ln \left(\frac{J_s(\Theta_1)}{I_s(\Theta_1)} \right) - \ln \left(\frac{J_s(\Theta_0)}{I_s(\Theta_0)} \right) = D_k(s) - D_l(s)$$

This last equation allows us to compute the change in aggregate TFP following the generalization of the experiment:

$$\Delta \ln(TFP) = \sum_{s=1}^S \phi_s \frac{1 - \theta_s}{\theta_s} D_l(s) - \left(\sum_{s=1}^S \alpha_s \phi_s \right) \ln \left(\sum_{s=1}^S \frac{K_s}{K} e^{(D_k(s) - D_l(s))} \right),$$

where $\frac{K_s}{K}$ is the share of industry s in the aggregate capital stock.

Under the parametric assumptions **16**, we have that:

$$\ln \left(\frac{J_s(\Theta_1)}{I_s(\Theta_1)} \right) - \ln \left(\frac{J_s(\Theta_0)}{I_s(\Theta_0)} \right) = -\Delta \mu_\tau + \Delta \sigma_\tau^2 \left(\frac{1}{2} + \frac{\alpha \theta}{1 - \theta} \right) - \frac{\theta}{1 - \theta} \Delta \sigma_{\tau z}$$

Therefore, change in aggregate TFP following the extension of the treatment to all firms in the economy becomes:

$$\begin{aligned} \Delta \ln(TFP) = & - \sum_{s=1}^S \alpha_s \phi_s \left(D_{lMRPK}(s) + \frac{1}{2} \frac{\alpha_s \theta_s}{1 - \theta_s} DV_{lMRPK}(s) + DCOV(s) \right) \\ & - \left(\sum_{s=1}^S \alpha_s \phi_s \right) \ln \left(\sum_{s=1}^S \frac{K_s}{K} e^{-D_{lMRPK}(s) + \frac{1}{2} DV_{lMRPK}(s) - DCOV(s)} \right), \end{aligned}$$

B.6 Proof of Proposition 7

We first show that Proposition 1 still holds with decreasing returns to scale.

With monopolistic competition and decreasing returns to scale, for a firm i with a stock of capital k_{it} , operating profits after optimizing labor demand are given by:

$$\begin{aligned} p_{it}y_{it} - w l_{it} &= (1 - (1 - \alpha)\nu\theta) \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} Y^{\frac{1-\theta}{1-(1-\alpha)\nu\theta}} e^{z_{it} \frac{\theta}{1-(1-\alpha)\nu\theta}} k_{it}^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}} \\ &= S_t \times (1 - (1 - \alpha)\nu\theta) ((1 - \alpha)\nu\theta)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} e^{z_{it} \frac{\theta}{1-(1-\alpha)\nu\theta}} \left(\frac{k_{it}}{S_t} \right)^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}}, \end{aligned}$$

where $S_t = \frac{Y^{\frac{1-\theta(1-\alpha\nu)}{1-(1-\alpha)\nu\theta}}}{w^{\frac{\nu\theta}{1-(1-\alpha)\nu\theta}}}$.

It follows directly from the proof of Proposition 1 that in this economy, and under the assumptions of Proposition 1, the ergodic joint distribution of capital wedges and productivity is independent of (w, Y) and depend only on the parameters Θ . Let $F(z, \tau; \Theta)$ denote this distribution as before.

With decreasing returns to scale ν , profit maximization for firm i in industry s as a function of a capital wedge τ_{is} leads to:

$$\begin{cases} k_{it} = \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{\nu(1-\alpha)\theta}{1-\nu\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_{it} \frac{\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r(1 + \tau_{it})} \right)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}} \\ l_{it} = \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{1-\alpha\nu\theta}{1-\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_{it} \frac{\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r(1 + \tau_{it})} \right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} \end{cases}$$

Firm i output at the optimum is given by:

$$p_{it}y_{it} = Y^{1-\theta} e^{\theta z_{it}} (k_{it}^\alpha l_{it}^{1-\alpha})^{\nu\theta} = Y^{\frac{1-\theta}{1-\nu\theta}} \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{(1-\alpha)\nu\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r(1 + \tau_{it})} \right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} e^{z_{it} \frac{\theta}{1-\nu\theta}} \quad (21)$$

And the firm's production writes:

$$y_{it}^\theta = Y^{\frac{(1-\theta)\nu\theta}{1-\nu\theta}} \left(\frac{(1 - \alpha)\nu\theta}{w} \right)^{\frac{(1-\alpha)\nu\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r(1 + \tau_{it})} \right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} e^{z_{it} \frac{\theta}{1-\nu\theta}} \quad (22)$$

Equilibrium on the product market implies $Y^\theta = \int_i y_i^\theta di$, which is equivalent to:

$$\frac{w}{\bar{w}} = \left(\frac{(1-\alpha)\nu\theta}{\bar{w}} \right) \left(\frac{\alpha\nu\theta}{r} \right)^{\frac{\alpha}{1-\alpha}} Y^{-\frac{1-\nu}{(1-\alpha)\nu}} \left(\int_{(z,\tau)} \frac{e^{z_{it}\frac{\theta}{1-\nu\theta}}}{(1+\tau_{it})^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{1-\nu\theta}{(1-\alpha)\nu\theta}} \quad (23)$$

Equilibrium on the labor market implies that:

$$Y = \bar{L} \left(\frac{w}{\bar{w}} \right)^{1+\epsilon} \left(\frac{(1-\alpha)\nu\theta}{\bar{w}} \right)^{-1} \quad (24)$$

Combining these two equations provides aggregate output:

$$\frac{Y^{1+\frac{(1+\epsilon)(1-\nu)}{(1-\alpha)\nu}}}{\bar{L}} = \left(\frac{(1-\alpha)\nu\theta}{\bar{w}} \right)^\epsilon \left(\frac{\alpha\theta\nu}{r} \right)^{\frac{\alpha}{1-\alpha}(1+\epsilon)} \left(\int_{(z,\tau)} \frac{e^{z_{it}\frac{\theta}{1-\nu\theta}}}{(1+\tau_{it})^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{(1+\epsilon)(1-\nu\theta)}{(1-\alpha)\nu\theta}}, \quad (25)$$

which implies:

$$\begin{aligned} \frac{Y}{\bar{L}^{\frac{(1-\alpha)\nu}{(1-\nu)(1+\epsilon)+(1-\alpha)\nu}}} &= \left(\frac{(1-\alpha)\nu\theta}{\bar{w}} \right)^{\frac{\nu(1-\alpha)\epsilon}{(1-\nu)(1+\epsilon)+(1-\alpha)\nu}} \left(\frac{\alpha\theta\nu}{r} \right)^{\frac{\alpha\nu(1+\epsilon)}{(1-\nu)(1+\epsilon)+(1-\alpha)\nu}} \\ &\quad \times \left(\int_{(z,\tau)} \frac{e^{z_{it}\frac{\theta}{1-\nu\theta}}}{(1+\tau_{it})^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{(1+\epsilon)(1-\nu\theta)}{\theta((1-\alpha)\nu+(1+\epsilon)(1-\nu))}} \end{aligned}$$

Finally, aggregate TFP admits a simple expression:

$$TFP = \frac{Y}{K^{\alpha\nu} L^{(1-\alpha)\nu}} = \left(\int_{(z,\tau)} \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{\frac{1-(1-\alpha)\nu\theta}{\theta}} \left(\int_i \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta) \right)^{-\alpha\nu}$$

With decreasing returns to scale ν , empirically, we have:

$$\begin{aligned} \mathbb{E}[l_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)] &= \mathbb{E}[l_{it}|\Theta_{it} = \Theta_1; (\bar{w}, Y_0)] \\ &= \left(\frac{(1-\alpha)\nu\theta}{w} \right)^{\frac{1-\alpha\nu\theta}{1-\theta}} Y_0^{\frac{1-\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r} \right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} \int_{(z,\tau)} \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_1), \end{aligned}$$

Therefore, the sufficient statistics D_l takes the following form:

$$\begin{aligned}
D_l &= \ln(\mathbb{E}[l_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[l_{it}|T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \\
&= \ln(\mathbb{E}[l_{it}|\Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[l_{it}|\Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \quad \text{as treatment is randomized} \\
&= \ln \left(\int_{(z,\tau)} \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_1) \right) - \ln \left(\int_{(z,\tau)} \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_0) \right)
\end{aligned}$$

And we see directly that:

$$\Delta \ln(Y) = \frac{(1+\epsilon)(1-\nu\theta)}{\theta((1-\alpha)\nu + (1+\epsilon)(1-\nu))} D_l$$

To obtain an expression for the change in aggregate TFP in the counterfactual economy, we start from:

$$\begin{aligned}
\mathbb{E}[k_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)] &= \mathbb{E}[k_{it}|\Theta_{it} = \Theta_1; (\bar{w}, Y_0)] \\
&= \left(\frac{(1-\alpha)\nu\theta}{w} \right)^{\frac{\nu(1-\alpha)\theta}{1-\nu\theta}} Y_0^{\frac{1-\theta}{1-\nu\theta}} \left(\frac{\alpha\nu\theta}{r} \right)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}} \int_i \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_1)
\end{aligned}$$

Therefore, the estimator D_k corresponds to:

$$\begin{aligned}
D_k &= \ln(\mathbb{E}[k_{it}|T_{it} = 1, \Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[k_{it}|T_{it} = 0, \Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \\
&= \ln(\mathbb{E}[k_{it}|\Theta_{it} = \Theta_1; (\bar{w}, Y_0)]) - \ln(\mathbb{E}[k_{it}|\Theta_{it} = \Theta_0; (\bar{w}, Y_0)]) \quad \text{as treatment is randomized} \\
&= \ln \left(\int_i \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_1) \right) - \ln \left(\int_i \frac{z_i^{\frac{\theta}{1-\nu\theta}}}{(1+\tau_i)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z, \tau; \Theta_0) \right)
\end{aligned}$$

And we can thus express the change in aggregate TFP resulting from a generalization of the experiment to all firms in the economy as:

$$\Delta \ln(TFP) = \frac{1 - (1-\alpha)\nu\theta}{\theta} D_l - \alpha\nu D_k$$

If we make the parametric assumptions 10, then we have:

$$\Delta \ln Y = -\frac{(1+\epsilon)(1-\nu\theta)}{\theta((1-\alpha)\nu + (1+\epsilon)(1-\nu))} \frac{\alpha\nu\theta}{1-\nu\theta} \left(\Delta\mu_\tau - \frac{1}{2} \frac{\theta}{1-\nu\theta} (\alpha\nu\Delta\sigma_\tau^2 - 2\Delta\sigma_{\tau z}) \right)$$

We see that with decreasing returns to scale ν : $D_{IMRPK} = \Delta\mu_\tau$, $DV_{IMRPK} = \Delta\sigma_\tau^2$ and $DCOV = \frac{\theta}{1-\nu\theta}\Delta\sigma_{\tau z} - \frac{\alpha\nu\theta}{1-\nu\theta}\Delta\sigma_{\tau z}$, so that:

$$\Delta \ln Y = -\frac{\alpha\nu(1+\epsilon)}{(1-\alpha)\nu + (1+\epsilon)(1-\nu)} \left(D_{IMRPK} + \frac{1}{2} \frac{\alpha\nu\theta}{1-\nu\theta} DV_{IMRPK} + DCOV \right)$$

And the expression for the change in aggregate TFP following a generalization of the experiment becomes:

$$\Delta \ln(TFP) = -\frac{1}{2} \alpha\nu \frac{1 - (1-\alpha)\nu\theta}{1-\nu\theta} DV_{IMRPK}$$