Preliminary and Incomplete

Credit Ratings: Strategic Issuer Disclosure and Optimal Screening*

Jonathan Cohn† Uday Rajan‡ Günter Strobl§

March 25, 2018

Abstract

We study a model in which an issuer can manipulate information obtained by a credit rating agency (CRA) seeking to screen and rate its financial claim. Better CRA screening leads to a lower probability of obtaining a high rating but makes a high rating more valuable. Over an intermediate range of manipulation cost, improving screening quality can lead to more manipulation, dampening the CRA's incentive to screen. We further show that a CRA's own incentives to inflate ratings constrain its optimal screening intensity. Our model suggests that strategic disclosure by issuers may have played a role in recent ratings failures.

*This paper is a substantially revised version of a previous paper with the same title. We thank Andres Almazan, Aydoğan Altı, Taylor Begley, Sugato Bhattacharyya, Patrick Bolton, Matthieu Bouvard, Sivan Frenkel, John Griffin, David Hirshleifer, Joel Shapiro, Christian Opp, Marcus Opp, Amiyatosh Purnanandam, David Scharfstein, Martin Schmalz, and Sheridan Titman, and participants at seminars at Boston College, Boston University, Duke, Georgia Tech, IDC Herzliya, Indiana, Michigan, NYU, Tel Aviv, UT Austin, UC Davis, UC Irvine, and the Credit Ratings, EFA, FIRS, Erasmus Credit, Oxford Financial Intermediation, and LSE Paul Woolley conferences for helpful comments. Errors and omissions remain our responsibility.

†McCombs School of Business, University of Texas, Austin; jonathan.cohn@mccombs.utexas.edu

‡Stephen M. Ross School of Business, University of Michigan, Ann Arbor; urajan@umich.edu

§Frankfurt School of Finance and Management; g.strobl@fs.de

1 Introduction

The failure of credit ratings to predict defaults of mortgage-backed securities in the lead-up to the financial crisis raises questions about the role of credit rating agencies (CRAs) in the economy. In principle, these "information intermediaries" create value by producing information in the form of ratings that allows investors to more accurately price assets. However, incentive problems may distort ratings and hence their usefulness. For example, there is evidence that CRAs inflated their ratings of mortgage-backed securities pre-crisis, allowing them to obtain more fees from issuers. An under-explored facet of the problem is that CRAs rely on issuers for much of the information on which they base their ratings, and issuers have incentives to distort this information in an effort to obtain more favorable ratings.

This paper investigates the implications of issuers' ability to distort information used to rate securities by modeling a game of costly screening and costly signal manipulation between a CRA and issuer. If the issuer in the model has a low-quality asset to sell, he can attempt, at a cost, to induce a favorable rating by manipulating the CRA's information. At the same time, the CRA chooses how much to invest in screening (its "screening intensity") to reduce the probability that such manipulation is successful.

We show that these two simple decisions interact in complex ways. The CRA, which places weight on rating accuracy, always increases its screening intensity when it assigns higher probability to the issuer manipulating. However, the issuer may manipulate more rather than less frequently when he expects the CRA to screen more intensely. As a result, a reduction in the cost of screening (or an increase in the weight the CRA places on ratings accuracy), though it leads to greater screening intensity, may have little effect on ratings accuracy. We also show that the CRA may not increase its screening intensity in response to a fall in overall asset quality, even though it views rating errors as costly. In addition, if the CRA and issuer bargain over the fee for a rating upfront and the issuer's incentives to manipulate are very strong, the CRA may actually screen more intensely when the issuer's cost of manipulation increases.

Many commentators have argued that CRAs were at least negligent and possibly complicit with issuers in the build-up to the financial crisis. We are by no means attempting to diminish the culpability of the CRAs. However, our model puts some focus back on the role of the issuer in misleading both CRAs and investors.[1] Our results suggest that the lack of CRA response

[1] In a written statement before the US SEC on November 21, 2002, Raymond W. McDaniel, President of

to a fall in overall asset quality is not itself conclusive evidence of CRA malfeasance. They also suggest the need to account for issuer behavior when using observed ratings accuracy to assess CRA diligence. Finally, penalizing issuers for distorting information can increase ratings accuracy by improving CRA screening incentives even if these penalties are too small to affect issuer behavior.

To be more precise about the details of the model, the issuer has either a high-quality (positive NPV) or low-quality (negative NPV) project that he wishes to sell to investors as a security. For convenience, we also refer to the issuer himself as the high or low type, depending on his project quality. The issuer begins by deciding whether or not to apply for a rating. The CRA subsequently observes a high or low signal of project quality and assigns the security a high or low rating consistent with its signal.[2] The issuer pays a rating fee to the CRA if the CRA assigns a high rating and then sells his security to investors, who form rational expectations and break even in expectation.[3] Finally, the project pays off. We first treat the rating fee as exogenous and then endogenize it as the solution to a Nash bargaining game played before the issuer learns his type.

If the project is high quality, the CRA observes a high signal. If the project is low quality, it may observe either a high or low signal, with probabilities depending on two simultaneous decisions made before the CRA draws the signal. The low-type issuer decides whether or not to take a costly, unobserved action that increases the likelihood that the CRA (incorrectly) draws a high signal. We refer to this action as manipulation of the CRA's signal. The cost of this action, which we term a "manipulation cost," captures the direct costs of distorting information as well as expected longer-run costs associated with sanctions for false disclosure and loss of reputation.[4] At the same time, the CRA chooses how much to invest in screening in order to reduce the likelihood that manipulation is successful.

Given the information structure of the game, either a low rating or a lack of rating reveal

Moody's Investors Service, states "Most issuers operate in good faith and provide reliable information to the securities markets, and to us. Yet there are instances where we may not believe that the numbers provided or the representations made by issuers provide a full and accurate story."

[2]We assume that the CRA issues ratings faithful to its signal. While consideration of strategic rating inflation is beyond the scope of this paper, the CRA can still implicitly inflate ratings in the model through a lack of diligent screening. The discrete nature of the CRA's rating in our model is consistent with the actual ratings process. Goel and Thakor (2015) rationalize coarse (i.e., discrete) ratings. We assume that asset types and ratings are binary for simplicity. The general logic of the model would hold with more than two types and ratings, as long as issuers prefer higher ratings over lower ones.

[3]The assumption that the issuer pays only for a favorable rating is not crucial for the conclusions of the paper.

[4]In a previous version of the paper, we modeled the reputational consequences of manipulation explicitly by adding a second period of issuance and allowing issuers to develop a reputation for honest disclosure. The broad insights from that version of the model are similar to those we present here.

the issuer's security to be low quality. As a low-quality project is negative NPV, the issuer never sells an unrated or low-rated security. The price of a high-rated asset is the expected value of the project, given the equilibrium strategies of the issuer and CRA. A high-type issuer always requests a rating since he is guaranteed to receive a high rating. The low-type issuer may or may not request a rating, but he always manipulates if he requests a rating.[5] The issuer's payoff is the price of a high-rated asset if it sells one less the rating fee, less any manipulation costs. The CRA's payoff is the fee for a high rating if it issues one, less a penalty for a rating error if it commits one, less the cost of screening.

We show that there are three types of possible equilibria in the game, depending on the cost of manipulation (relative to the other parameters of the model). When the cost is high, the issuer never manipulates. Anticipating this, the CRA invests zero in screening. When the manipulation cost is low, the low-type issuer always manipulates, and the CRA invests heavily in screening. When the manipulation cost is moderate, the low-type issuer mixes between manipulating and not manipulating, and the CRA invests a moderate amount in screening. We refer to these outcomes as "zero-manipulation," "full-manipulation," and "partial-manipulation" equilibria, respectively. Most of the analysis focuses on the partial-manipulation equilibrium.

The key to many of our conclusions lies in the low-type issuer's response to anticipated changes in CRA screening intensity. More intense anticipated screening lowers his assessment of the probability that he can sell a low-rated asset with a high rating attached. This direct "filtering" effect of increased screening intensity discourages manipulation. However, if investors also anticipate more intense screening, then they perceive a high rating as more credible and therefore price a high-rated project more favorably. This anticipated "price improvement" effect of increased screening intensity encourages manipulation.

The filtering effect of an increase in anticipated screening intensity dominates when this intensity is already high (e.g., when the CRA's screening cost is low or the penalty for a rating error is high), but the price improvement effect dominates when anticipated screening intensity is low. The low-type issuer's expected benefit from manipulation is the price of a high-rated security times the probability that manipulation succeeds. When anticipated screening intensity is high, the price of a high-rated security is high, magnifying the negative incentive effect of a reduction in the probability that manipulation succeeds. In contrast,

[5] The CRA always assigns the low-type's project a low rating if the issuer does not manipulate, and investors infer that a security for which the issuer does not seek a rating must be low quality, so not manipulating is equivalent to not requesting a rating.

when anticipated screening intensity is low, the probability of manipulation success is high, magnifying the positive incentive effect of a price improvement.

This non-monotonicity in issuer response to anticipated screening intensity leads to somewhat counterintuitive conclusions about issuer behavior in a partial-manipulation equilibrium. Manipulation increases with the cost of CRA screening when that cost is low, but actually decreases with the cost of CRA screening when the cost is high. Similarly, manipulation decreases with the CRA's rating error penalty when that penalty is large, but actually increases with the penalty when the penalty is high. From a policy standpoint, the issuer's response can either magnify or weaken the impact on rating accuracy of efforts to encourage stricter CRA screening.

The model also produces an interesting conclusion about the optimal response of CRA screening policy to changes in average asset quality. Holding fixed the issuer's behavior, the CRA responds to a reduction in average project quality (lower proportion of good projects) by intensifying screening, as one would expect. However, in a partial-manipulation equilibrium, reduced average project quality weakens the low-type issuer's incentive to manipulate, as there are fewer high-quality projects with which to potentially pool by manipulating. We show that the issuer's response perfectly offsets the direct effect of lower average project quality on CRA screening incentives, and the optimal screening level remains unchanged.

Finally, we obtain one insight from analyzing the full-manipulation equilibrium case. In this corner solution, a small increase in the issuer's manipulation cost has no effect on its behavior. He continues to manipulate with probability one when he has a low-quality asset. When the rating fee is exogenous, the CRA's payoff — and hence optimal screening intensity — only changes with manipulation cost if the issuer's strategy changes. However, when the issuer and CRA bargain over the rating fee ex ante, the issuer passes part of the higher expected manipulation cost through to the CRA in the form of a lower equilibrium rating fee. This lower fee reduces the benefit of assigning a high rating to a low-quality asset, leading to higher optimal screening intensity.

Our paper contributes primarily to the still-burgeoning literature examining the process by which CRAs produce information and assign credit ratings to issuers. Most of the papers in this literature focus on distortions in the ratings process due to the ability of issuers to "shop for favorable ratings (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012; Sangiorgi and Spatt, 2015) or CRA incentives to inflate (Frenkel, 2015; Bouvard and Levy, 2013; Fulghieri, Strobl, and Xia, 2014) ratings in order to increase fee income. Our paper adds to this literature by accounting for the ability of an issuer to distort the information on which a CRA relies in evaluating the issuers securities and assigning ratings. Our results demonstrate that a more stringent ratings process can either mitigate or amplify these incentives. The issuers response may therefore either enhance or undermine efforts to improve ratings quality by lowering the cost of screening or holding CRAs more accountable for ratings errors.[6]

Our paper also contributes to the long literature examining information manipulation. Stein (1989), Goldman and Slezak (2006), and Crocker and Slemrod (2007), among others, show that concerns about short-term stock price performance can lead to "signal-jamming equilibria in which managers take costly actions to manipulate their performance reports, even though investors undo this manipulation in equilibrium. Strobl (2013) demonstrates that more informative accounting disclosures can actually lead to more earnings manipulation by making investors inference about firms with high reported earnings more favorable. Song and Thakor (2006) show that a CEO may withhold information from a board of directors to increase the probability of approval of her favored projects. Our paper augments this class of models by adding a certifying agent and considering optimal screening policy in the face of potential information manipulation.[7]

Our paper is also related to the economics of crime literature. Tsebelis (1990) argues that an enforcement agency optimally responds to an increase in the penalty for crime by weakening enforcement, undoing the effect of the increased penalty.[8] A similar phenomenon generally occurs in our setting, with the CRA optimally reducing screening intensity in response to an increase in the cost of manipulation. However, as noted, an increase in the cost of manipulation can actually result in greater screening intensity when the issuer's incentives to manipulate are overwhelming and the issuer and CRA bargain over a rating fee upfront. We also add to this literature by exploring the effects of the cost of enforcement in the form of CRA screening, showing that an increase in this cost has ambiguous effects on the extent of issuer misbehavior.

Finally, our paper contributes to the set of papers demonstrating that having more in-

[6]While we focus on credit ratings, one can imagine similar effects at work in audit settings as long as the audit is stochastic, as in Mookherjee and Png (1989). Of course, an important difference in our model, compared to many papers in this setting (e.g., Townsend (1979)) is that it is generally assumed that the principal can commit to the frequency of an audit. A notable exception is (Khalil, 1997).

[7]Gill and Srgoi (2012) analyze a model where a seller can choose the stringency of a certification test. In some ways similar to our setting, greater stringency hurts the seller by reducing the probability of certification but benefits the seller by increasing the price conditional on certification. However, there is no manipulation in their model.

[8]Cox (1994) shows that the Tsebelis result is not robust when agents have a heterogeneous benefit from committing a crime.

formation about an agent can prove counterproductive. More information about an agents innate skill can dampen her incentives to work hard in order to prove her worth (e.g., Holmström, 1999; Dewatripont, Jewitt, and Tirole, 1999). More information about an agents actions can induce the agent to disregard her own information and to instead "herd with more talented agents (Prendergast, 1993; Brandenburger and Polak, 1996; Prat, 2005). Cohn and Rajan (2013) show that more information-gathering by a board of directors about a CEO's skill exacerbates agency problems caused by the CEO's reputational concerns. In our model, the issuers response to the CRAs anticipated investment in information about project quality can mute the positive effects of that investment on ratings accuracy. When that is the case, the CRA would naturally benefit if it could somehow commit to a lower level of signal precision than it will choose in equilibrium.

2 Model

An issuer has a project that requires an upfront investment and generates a future cash flow. The project is high quality with probability η and low quality with probability $1-\eta$. The high-quality project has an NPV $v_h > 0$, and the low-quality project has an NPV $v_\ell < 0$. The values of η, v_h, and v_ℓ are common knowledge.

The issuer privately observes the quality of its own project. The issuer has no financial resources, so if it wishes to undertake the project, it must issue a financial claim backed by the project. If it does not undertake the project, it obtains a payoff of zero. For convenience, we refer to an issuer with a high- (low-) quality project as a high- (low-) type issuer. We normalize v_h to 1 and v_ℓ to -1. The average NPV of the project then is $2\eta - 1$. We assume that $\eta \geq \frac{1}{2}$, so the average project has a positive NPV.

An issuer wishing to sell a financial claim can obtain a rating from a credit rating agency (CRA). The CRA obtains a noisy signal about the type of the project and issues a rating. Investors observe the rating and decide whether they wish to buy the financial claim being issued, and, if so, at what price. Investors have no independent information about the type of the issuer's project. Based on the strategies of issuer and CRA and on the observed rating, they update their beliefs about the type of the project. Investors are risk-neutral and perfectly competitive, so if they purchase the claim, they do so at the expected NPV of the project, given their posterior beliefs.

There are two dates in the model, 0 and 1. The discount rate is normalized to zero. The sequence of events at date 0 is as follows:

1. The issuer privately observes its type, high or low.

2. Simultaneously,

   (i) The issuer chooses whether or not to approach the credit rating agency (CRA) to request a rating. If a low-type issuer requests a rating, it further chooses whether or not to take an unobserved action that we refer to as "manipulating" the information observed by the CRA (more on the consequences of this decision shortly). Manipulation incurs a cost $m > 0$, which captures both upfront costs of distorting or misrepresenting information presented to the CRA as well as expected longer-run costs associated with sanctions for false disclosure and loss of reputation.

   (ii) The credit rating agency (CRA) privately chooses a screening intensity α at a cost $kc(\alpha)$, where $k > 0$ and $c(\cdot)$ is strictly increasing and strictly convex. We assume that $c(0) = c'(0) = 0$, so that in the absence of screening, both total and marginal costs to the CRA are zero. We also assume that $\lim_{\alpha \to 1} c'(\alpha) = \infty$, which ensures that the equilibrium screening intensity is less than 1.

3. The CRA observes a binary signal of project quality. If project quality is high, then the CRA observes a high signal). If project quality is low and the issuer did not manipulate the information observed by the CRA, the CRA observes a low signal. If project quality is low and the issuer did manipulate the CRA's information, then the CRA observes a low signal with probability α and a high signal with probability $1 - \alpha$.

   The CRA then assigns a public rating, r_h ("high" rating) or r_ℓ ("low" rating), to the financial claim backed by the project. A CRA wishing to engage in rating inflation can do so more profitably by reducing its screening intensity, so at this stage we assume that the CRA commits to issue a rating that is faithful to its signal (i.e., the rating is r_h if its signal is high and r_ℓ if its signal is low).

   If the CRA assigns the claim a high rating, the issuer pays a fee $f \geq 0$ to the CRA.[9] We take this fee as exogenously given in Section 3 and later endogenize it as the outcome of a bargaining process in Section 4. We assume that this fee is not so large that it deters all issuance.

[9] As is standard in the literature, we assume that the issuer only pays the rating fee f if the rating is high. This assumption is consistent with the large literature that has commented on the incentive to inflate ratings in an issuer-pay model. Qualitatively, many of our results go through if the fee is paid upfront, before the rating is issued. Of course, the exact forms of the expressions we exhibit will change.

4. Investors compute the NPV of the project, given the CRA's report (r_h or r_ℓ), their beliefs about screening intensity α and about the reporting strategy of each type. From an investor's perspective, the financial claim is an asset. By construction, the high-rated asset has a weakly positive NPV, so sells at a price equal to its NPV. Conversely, the low-rated asset has a negative NPV, and so does not sell. If the security is issued, the project is carried out.

If the project is undertaken, at date 1 all parties find out the type of the project and earn their respective terminal payoffs. The CRA incurs a penalty λ if its rating proves to be incorrect; that is, if a claim with a rating of r_h proves to be backed by a low-quality project.[10] This penalty captures, in reduced form, the reputational cost of reduced investor confidence as well as explicit regulatory sanctions and losses from investor lawsuits.[11]

We consider stable perfect Bayesian equilibria of the game. Formally, there are three players to the game: the issuer, the CRA, and investors as a group. Other than the type of the firm and the actions of the issuer and CRA, all information is common knowledge.

Investors form rational expectations and compete in a perfectly competitive market, so earn zero profits in expectation. As noted, the issuer never sells a security receiving a low rating (or no rating) in equilibrium. The price of a high-rated security, p, equals the expected payoff of the project backing it, given investors' conjectures regarding α and σ. In equilibrium, these conjectures must be correct.

We restrict attention to equilibria that survive the $D1$ refinement. In such equilibria, a high-type issuer always requests a rating.[12] Thus, in describing equilibrium issuer strategies, we need only specify the strategy of a low-type issuer. In principle, a low type issuer makes two choices: whether or not to request a rating and, conditional on requesting a rating, whether or not to manipulate the CRA's information. However, requesting a rating but not manipulating guarantees the issuer a low rating, which reveals the issuer to be the low type and is therefore informationally equivalent to not requesting a rating. Therefore, to simplify

[10] In principle, a rating would also be incorrect if a claim with a rating of r_ℓ proves to be backed by a high-quality project. However, only upward rating errors are possible in our model. In that sense, an incorrect rating in our model is "inflated" in the sense that it induces a favorable belief about a low-quality security.

[11] As an example of these explicit costs, in 2015, Standard & Poor's reached a settlement with the Justice Department over inflated ratings on mortgage-backed securities over the period 2004–2007, and agreed to pay $1.375 billion.

[12] Absent the $D1$ refinement, there always exists an equilibrium in which no issuer requests a rating, supported by investors' beliefs attributing a deviation to the low-type issuer. $D1$ effectively requires that investors attribute a deviation to the type that faces a lower cost of deviating. The high type faces a lower cost of requesting a rating, since the low type must bear the manipulation cost m in order to have a chance of receiving a high rating and issuing a security.

the strategy space, we disallow the possibility that the low-type issuer can request a rating but not manipulate.

Thus, a low-type issuer may request a rating (and manipulate), not request a rating, or mix between the two actions. We denote by σ the probability with which the low-type issuer requests a rating. The only choice the CRA makes in the game is its level of screening intensity α. Thus, when the rating fee f is exogenous, a perfect Bayesian equilibrium of the game is represented by (α^*, σ^*), where each player's strategy maximizes its own payoff, given its own beliefs and the actions of other parties in the game. When f is endogenous, it too must be determined as part of the equilibrium.

We analyze this model in the next two sections. In Section 3, we take the rating fee f as fixed and solve for the equilibrium (α, σ). Much of the intuition in the paper comes from analyzing the fixed fee case. In Section 4, we endogenize the rating fee as the solution to a Nash bargaining game and solve for the equilibrium, (f, α, σ). We show that in equilibrium the fee is strictly less than the price of the high-rated security in this case, again ensuring that the high-type issuer always seeks a rating.

3 Fixed Rating Fee

In this section, we take the fee f as exogenous and fixed, and solve for the equilibrium (α, σ). We do so in two steps. First, we analyze the best responses of the CRA and low-type issuer, taking the price of a rated security, p, as given. We then close the model by imposing the requirement that p be set so that investors break even in expectation, and solve for the equilibrium.

3.1 Best responses of CRA and issuer

Consider the CRA's best response, taking the low-type issuer's strategy σ as given. Let $\tilde{\sigma}$ denote the CRA's conjecture of σ. The CRA's expected payoff depends in part on the quality of the pool of issuers it expects to request a rating. Define $\nu(\sigma) = \frac{\eta}{\eta+(1-\eta)\sigma}$ as the probability that the issuer is a high type, conditional on requesting a rating. Note that ν decreases with σ and can take on values between η (if $\sigma = 1$; that is, if all issuers request a rating) and 1 (if $\sigma = 0$; that is, if only high-type issuers request a rating). The total mass of issuers requesting a rating is given by $N(\tilde{\sigma}) = \eta + (1-\eta)\tilde{\sigma}$. Thus, given $\tilde{\sigma}$, the CRA's expected payoff is

$$\Psi(\tilde{\sigma}) = N(\tilde{\sigma})\Big\{\nu(\tilde{\sigma})f + [1-\nu(\tilde{\sigma})](1-\alpha)(f-\lambda) - kc(\alpha)\Big\}. \tag{1}$$

The CRA's objective is to choose α to maximize Ψ. Observe that as the CRA takes the low-type's strategy as given, maximizing Ψ is equivalent to maximizing the term in the parentheses in equation (1). The following lemma characterizes the CRA's best response. Proofs of all results are in Appendix A.

Lemma 1. *The best response of the CRA satisfies* $c'(\alpha) = [1 - \nu(\tilde{\sigma})] \max\left\{\frac{\lambda - f}{k}, 0\right\}$.

When $f < \lambda$, the CRA's optimal screening intensity increases with $\tilde{\sigma}$, as $\nu(\cdot)$ is a decreasing function. Intuitively, since the CRA views erroneous ratings as costly, it screens more intensely when it expects the low-type issuer to manipulate with higher probability. It also screens more intensely when the cost it ascribes to an erroneous rating, λ, is higher and less intensely when the fee it receives for rating a security, f, is higher.[13]

Next, consider the low-type issuer's best response, taking the CRA' strategy α and the anticipated price of a high-rated security p as given. Let $\tilde{\alpha}$ denote the low-type issuer's conjecture of α and $\tilde{p}$ the expected price p of a high-rated security. If the low-type issuer does not request a rating, then its project is revealed to be low quality. Since a low-quality project is negative NPV, the issuer does not sell an asset in this case, and its payoff is zero. If, on the other hand, the low-type issuer requests a rating, it bears the manipulation cost m, and expects to receive a high rating with probability $1 - \tilde{\alpha}$. If it obtains a high rating, it pays a fee f to the CRA and sells its asset at an anticipated price $\tilde{p}$. If it obtains a low rating, its payoff is zero. Thus, the low-type issuer's expected payoff if it requests a rating is

$$\pi_\ell(\tilde{\alpha}, \tilde{p}) = (1 - \tilde{\alpha})(\tilde{p} - f) - m. \tag{2}$$

The following lemma characterizes the low-type issuer's best response.

Lemma 2. *It is a best response for the low-type issuer to request a rating (i.e., set* $\sigma = 1$*) if* $(1 - \tilde{\alpha})(\tilde{p} - f) \geq m$*, and to not request a rating (i.e., set* $\sigma = 0$*) if* $(1 - \tilde{\alpha})(\tilde{p} - f) \leq m$*.*

Note that if $(1 - \tilde{\alpha})(\tilde{p} - f) = m$, then any $\sigma \in [0, 1]$ represents a best response for the low-type issuer. Equilibria in which the low-type issuer mixes between seeking a rating and not doing so are of particular interest to us in the rest of the paper.

[13] One may note that when $\alpha^* = 0$, it is curious why the rating is still relevant at this point. One can rationalize the existence of the CRA even when $\alpha^* = 0$ by introducing a third type of issuer with a terrible project that has extremely negative NPV (say a Ponzi scheme) and assuming that the CRA adds value because it can perfectly screen out the terrible type, even if it cannot distinguish between the high and low type.

3.2 Price of a high-rated security

Next, we solve for the price of a high-rated security p, given investors' beliefs about the issuer's and CRA's strategies. To keep notation simple, we assume that investors' beliefs coincide with those of the CRA and the issuer (which will be true in equilibrium), so that $\tilde{\alpha}$ is the investors' conjecture of α and $\tilde{\sigma}$ their conjecture of σ. As investors are perfectly competitive and form rational expectations, the price p must equal the expected value of an asset, conditional on a high rating. The price, then, must satisfy

$$p(\tilde{\alpha},\tilde{\sigma}) = \frac{\nu(\tilde{\sigma})v_h + [1-\nu(\tilde{\sigma})](1-\tilde{\alpha})v_\ell}{\nu(\tilde{\sigma}) + [1-\nu(\tilde{\sigma})](1-\tilde{\alpha})} = \frac{\nu(\tilde{\sigma}) - [1-\nu(\tilde{\sigma})](1-\tilde{\alpha})}{\nu(\tilde{\sigma}) + [1-\nu(\tilde{\sigma})](1-\tilde{\alpha})}. \tag{3}$$

Observe that p is decreasing in $\tilde{\sigma}$ and increasing in $\tilde{\alpha}$. Holding $\tilde{\alpha}$ fixed, a higher anticipated probability of manipulation by the low-type issuer implies a worse pool of issuers requesting a high rating. As long as $\tilde{\alpha} < 1$, the pool of issuers that investors expect to survive screening and receive a high rating also worsens, resulting in a lower expected payoff from a high-rated asset, and hence a lower price. Similarly, holding $\tilde{\sigma}$ fixed, a higher anticipated screening intensity filters out more low-type issuers requesting ratings, improving the expected pool of high-rated assets and hence increasing the price investors are willing to pay for a high-rated asset.

3.3 Equilibrium

Now, let α^*, σ^*, and p^* denote the equilibrium values of α, σ, and p. Since all agents in the model form rational expectations, in equilibrium it must be that $\tilde{\alpha} = \alpha^*$, $\tilde{\sigma} = \sigma^*$, and $\tilde{p} = p^*$. Suppose that the CRA and investors conjecture that the low-type issuer always requests a rating (i.e., $\tilde{\sigma} = 1$). Then, $\nu = \eta$ and the CRA's best response is $\alpha_1 = c'^{-1}\left((1-\eta)\max\left\{\frac{\lambda-f}{k},0\right\}\right)$. Accounting for this response, investors set the price of high-rated security to $p(\alpha_1, 1)$. Given this price and a belief that $\alpha = \alpha_1$, it is a best response for the low-type issuer to request a rating if $m \leq (1-\alpha_1)(p(\alpha_1,1) - f)$. The best response is unique if $m < (1-\alpha_1)(p(\alpha_1,1) - f)$.

The conjecture $\tilde{\sigma} = 1$ represents the most pessimistic belief investors can hold about the type of issuer requesting a rating. Less pessimistic beliefs ($\tilde{\sigma} < 1$) would raise the price p and make requesting a rating more attractive. So, if the low-type issuer prefers to request a rating when $\tilde{\sigma} = 1$, it also prefers to request a rating for any lower value of $\tilde{\sigma}$. Thus, if the manipulation cost is sufficiently low, in equilibrium the low-type issuer always requests a rating, so that $\sigma^* = 1$. We call such an equilibrium a "full-manipulation" equilibrium.

Next, suppose that the CRA and investors conjecture that the low-type issuer never requests a rating (i.e., $\tilde{\sigma} = 0$). Then, $\nu = 1$, and the CRA's best response is to set $\alpha = 0$. Therefore, the price of a high-rated security is rationally set to 1. For the low-type issuer, in turn, it is a best response to not request a rating if $m \geq 1 - f$. As p is decreasing in $\tilde{\sigma}$, if the low-type issuer prefers to not request a rating when $\tilde{\sigma} = 0$, it always prefers to not request a rating. So, if the manipulation cost is sufficiently high, then the low-type issuer never requests a rating in equilibrium ($\sigma^* = 0$). We call such an equilibrium a "zero-manipulation" equilibrium.

For values of m such that $(1 - \alpha_1)(p(\alpha_1, 1) - f) < m < 1 - f$, no equilibrium in which the issuer plays a pure strategy ($\sigma^* = 0$ or $\sigma^* = 1$) is sustainable. In this case, the low-type issuer mixes between requesting and not requesting a rating. For such mixing to be sustainable, the low-type issuer must be indifferent between the two actions, so that (α^*, σ^*) must satisfy the condition $(1 - \alpha^*)(p(\alpha^*, \sigma^*) - f) = m$.

Note that α_1 and $p(\alpha_1, 1)$ are functions only of exogenous parameters. We can now fully characterize the equilibrium of the game.

Proposition 1. *When $f < 1$ is fixed, the equilibrium is as follows:*

(i) If $m \leq (1 - \alpha_1)(p(\alpha_1, 1) - f)$, the equilibrium features full manipulation, with $\alpha^* = \alpha_1$ and $\sigma^* = 1$.

(ii) If $m \geq (1 - f)$, the equilibrium features zero manipulation, with $\alpha^* = \sigma^* = 0$.

(iii) If $(1 - \alpha_1)(p(\alpha_1, 1) - f) < m < 1 - f$, the equilibrium features partial manipulation, with (α^*, σ^*) satisfying the two conditions

$$\sigma^* = \frac{\eta}{1 - \eta} \left(\frac{1 - f - \frac{m}{1-\alpha^*}}{(1 - \alpha^*)(1 + f + \frac{m}{1-\alpha^*})} \right), \quad \textit{and} \tag{4}$$

$$\alpha^* = c'^{-1} \left((1 - \nu(\sigma^*)) \max \left\{ \frac{\lambda - f}{k}, 0 \right\} \right). \tag{5}$$

The condition $f < 1$ ensures that, in equilibrium, the high-type issuer seeks a rating with probability 1. In equilibrium, either $\sigma^* = 0$, in which case $p(\alpha^*, \sigma^*) = 1 > f$, or $\sigma^* > 0$, in which case it must be that $p > f$. If $f \geq 1$, even when the low-type issuer is fully screened out, the high-type issuer weakly prefers to not seek a rating, which implies that no ratings will be issued.

Going forward, the partial-manipulation equilibrium is of particular interest to us. We establish sufficient conditions for uniqueness of such an equilibrium. As the parameter regions in which full-, partial-, and zero-manipulation equilibria do not overlap, under these conditions the overall equilibrium is also unique. The condition $f < \lambda$ in the next lemma ensures that $\alpha^* > 0$; that is, the screening intensity of the CRA is strictly positive in equilibrium.

Lemma 3. *Suppose $f < \lambda$ and let α^* denote the equilibrium screening intensity in a partial-manipulation equilibrium. The equilibrium is unique if*

$$\frac{c''(\alpha^*)}{c'(\alpha^*)} > \frac{1-f}{2}\left(\frac{1+f}{m} - \frac{\eta}{1-\eta}\right). \tag{6}$$

Although condition (6) relies on the endogenous value of α^*, for specific cost functions it is readily translatable to a function of the exogenous parameters. For example, suppose the cost function is $c(\alpha) = \alpha^q$ where $q > 1$. Further, suppose that $m < \frac{1-\eta}{\eta}(1+f)$, so that the RHS of condition (6) is strictly positive. Then, condition (6) reduces to $\alpha^* < (q-1)/\left(\frac{1-f}{2}\left(\frac{1+f}{m} - \frac{\eta}{1-\eta}\right)\right)$. When q is sufficiently high (i.e., the cost function is sufficiently convex), the RHS of the last inequality exceeds 1, so that the inequality is always satisfied.

For the rest of this paper, we assume that the equilibrium of the game is unique when the fee f is fixed; that is, we assume that the condition in Lemma 3 is satisfied in a partial-manipulation equilibrium.

In a partial-manipulation equilibrium, the equilibrium α^* and σ^* are jointly determined as the solution to the equations (4) and (5). Equation (5) can readily be interpreted as the CRA's optimal response to the equilibrium σ^*. The CRA increases its screening intensity, α^*, as σ^* increases. This response is expected, as increased manipulation by the low type worsens the pool of issuers requesting a rating.

The function on the right-hand side of equation (4) is more complex. Here, σ^* is non-monotone in α^*—for high values of α^*, σ^* decreases with α^*, while it actually increases with α^* for low values of α^*. Loosely, this equation may be interpreted as providing the optimal equilibrium response of the low-type issuer, taking into account that investors have rational conjectures over α and σ.

Intuitively, a change in expected screening intensity has two countervailing effects on the low-type issuer's incentives to request a rating. On the one hand, more intense screening lowers the probability of successful manipulation, which (given that the manipulation cost has stayed fixed) dissuades the low type from requesting a rating. This direct screening effect is large when screening intensity is high, because all else equal, a high α implies a high price for the security. On the other hand, in equilibrium more intense screening increases the price conditional on a high rating, encouraging the low type to request a rating. This indirect "price improvement" effect is large when screening intensity is low, since the probability that manipulation is successful is then high. These countervailing effects affect the response of α^* and σ^* to changes in the exogenous parameters.

3.4 Comparative statics

The exogenous parameters of the model are η, k, λ, m and (in this section) f. We now consider how small changes in each of these parameters affects the equilibrium (α^*, σ^*) in a partial-manipulation equilibrium. In addition to the equilibrium responses of the CRA and the issuer, both from an investor point of view and in terms of overall welfare, the proportion of low-type issuers that obtain a high rating, $\sigma^*(1-\alpha^*)$, is of interest. A lower proportion here implies a greater proportion of high-type issuers in the posterior (or post-screening) pool. In equilibrium, the price of a high-rated security is $p(\alpha^*, \sigma^*) = \frac{\nu(\sigma^*)-(1-\nu(\sigma^*))(1-\alpha^*)}{\nu(\sigma^*)+(1-\nu(\sigma^*))(1-\alpha^*)}$. Multiplying both numerator and denominator by $N(\sigma^*)$, the price may also be written as $p(\alpha^*, \sigma^*) = \frac{\eta-(1-\eta)\sigma^*(1-\alpha^*)}{\eta+(1-\eta)\sigma^*(1-\alpha^*)}$.

Thus, the price of a high-rated security is directly related to the quality of the posterior pool. The overall quality of the screening process is therefore directly captured by the price. Investment efficiency in this model is inversely related to the mass of low-type issuers that obtain the high rating, and so a higher price implies better investment outcomes.

Consider first the effect of an increase in η, the ex ante proportion of high-type issuers. Somewhat surprisingly, we show that both the level of screening and the price of the high-rated security are invariant to small changes in η in a partial-manipulation equilibrium. In our discussion of the comparative statics, we restrict attention to the case where $f < \lambda$, so that the screening intensity of the CRA is strictly positive.

Proposition 2. *Consider a partial-manipulation equilibrium with fixed $f < \lambda$. Then, $\frac{d\sigma^*}{d\eta} > 0$, $\frac{d\alpha^*}{d\eta} = 0$, and $\frac{dp(\alpha^*,\sigma^*)}{d\eta} = 0$. That is, as η increases, the CRA's screening intensity remains the same and the low-type issuer increases its manipulation intensity just enough to keep the price of the high-rated security unchanged.*

Consider the effect of a small increase in η. All else equal (in particular, holding σ^* fixed), an increase in η improves the pool of issuers requesting a rating (i.e., increases ν), weakening the CRA's incentive to screen. On the issuer's side, holding α^* fixed, an improvement in the pool of issuers requesting a rating increases the price conditional on a high rating, which strengthens the low-type issuer's incentive to request a rating. The low-type issuer responds by requesting a rating more frequently (increasing σ^*), degrading the pool of issuers requesting a rating (i.e., decreasing ν). In Proposition 2 we show that the low-type issuer's equilibrium response completely undoes the direct effect of an increase in η on the CRA's incentives to screen.

To see why this is the case, recall that in a partial-manipulation equilibrium, the low-type issuer must be indifferent between requesting and not requesting a rating. Further, if it seeks a rating, it must manipulate. Thus, the price p must satisfy $(1-\alpha^*)(p-f) = m$. Suppose that η increases by a small amount. By continuity, in the new equilibrium, the low-type issuer must continue to mix between seeking a rating and not. Observe that ν, the pool of issuers requesting a rating, is strictly increasing in η and strictly decreasing in σ. Thus, a small increase in η can be exactly offset by an increase in σ just large enough to ensure that ν remains constant. In turn, if ν is unchanged, the CRA's best response is unchanged. Finally, with ν and α^* unchanged, the price p remains constant as well. Thus, in the new equilibrium, σ^* increases just enough that the price of the high-rated security is unchanged, whereas α^* is unchanged.

Next, we consider the effects of a change in the cost of screening (k) and the penalty for a rating error (λ). These variables have opposite effects on the equilibrium screening intensity α^*. The conclusions for σ^* are less straightforward.

Proposition 3. *Consider a partial-manipulation equilibrium with fixed $f < \lambda$. Suppose that $(f+m)^2 + 2m < 1$. Then,*

(i) $\frac{d\alpha^*}{dk} < 0$. *Further, there exists a $\hat{k} > 0$ such that, if $k < \hat{k}$, then $\frac{d\sigma^*}{dk} > 0$, and if $k > \hat{k}$, then $\frac{d\sigma^*}{dk} < 0$. That is, as k increases, the low-type issuer manipulates more often when k is low and less often when k is high. In both cases, $\frac{dp(\alpha^*,\sigma^*)}{dk} < 0$; that is, the price of the high-rated security decreases.*

(ii) $\frac{d\alpha^*}{d\lambda} > 0$. *Further, there exists a $\hat{\lambda} > 0$ such that, if $\lambda < \hat{\lambda}$, then $\frac{d\sigma^*}{d\lambda} > 0$, and if $\lambda > \hat{\lambda}$, then $\frac{d\sigma^*}{d\lambda} < 0$. That is, as λ increases, the low-type issuer manipulates more often when*

λ is low and less often when λ is high. In both cases, $\frac{dp(\alpha^*,\sigma^*)}{d\lambda} > 0$; that is, the price of the high-rated security increases.

Not surprisingly, the CRA responds to an increase k, the cost of screening, or a decrease in λ, the penalty for a rating error, by reducing screening intensity. All else equal, one might expect that the low-type issuer would take advantage of weaker screening by manipulating more frequently. However, as already noted, the effect of a change in the anticipated level of α on σ may be ambiguous. In the proof of the proposition, we show that when $(f+m)^2+2m < 1$, indeed σ increases in α for small values of α, and decreases in α for large values of α.

When k is high or λ is low, the equilibrium screening level α^* is low. Therefore, the price of the high-rated security is relatively low. Further, a decrease in α^* in equilibrium results in an even lower price for the security. Trading off the higher likelihood of surviving screening (i.e., of getting a high rating) but the lower payoff conditional on survival (i.e., the lower price), the low-type issuer in fact manipulates less often in these cases (i.e., reduces σ^*).

Conversely, when k is low or λ is high, both α^* and the price of the high-rated security are relatively high. In these cases, the increased likelihood of surviving screening as α decreases dominates the price effect (i.e., the fact that the price falls as well). We then obtain the expected effect that σ^* increases; i.e., the low-type issuer manipulates more often.

The condition $(f+m)^2+2m < 1$ in the statement of the Proposition essentially requires the fee f and the manipulation cost m to be sufficiently low. When this condition is violated, σ unambiguously decreases in α. In this case, an increase in k leads to an increase in σ^*, and an increase in λ leads to a decrease in σ^*. As before, α^* decreases in k and increases in λ.

Next, consider changes in the manipulation cost, m. We show in Appendix B that, in a partial-manipulation equilibrium, α^* and σ^* both fall as the manipulation cost m increases. Here, the effects work in the expected direction. Further, as m increases, the price of the high-rated security increases; that is, the ex post pool of high-rated issuers improves in quality.

We note that, when the fee f is fixed, if the low-type issuer strictly prefers to manipulate in a full-manipulation equilibrium (i.e., an equilibrium in which $\sigma^* = 1$), small increases in m have no effect on either σ^* or α^*. If the manipulation cost increases by a small amount and the price of the high-rated security remains the same, the low-type issuer still strictly prefers to manipulate, so sets $\sigma^* = 1$. In turn, m can affect the CRA's first-order condition only through $\nu(\sigma^*)$, the conditional probability that an issuer applying for a rating is the high type. This probability does not change when σ^* is constant, so the issuer chooses the same value of α^* as before. In the next section, we note the contrast in this comparative static when the fee for a high rating, f, is endogenous rather than fixed.

Finally, consider a small increase in f, the fee for a high rating. We show in Appendix B that the screening intensity of the CRA, α^*, decreases when f increases. This is equivalent to rating inflation. In our model, the CRA has no incentive to set a high screening intensity and then misreport its signal, as it can obtain the exact same combination of fee revenue and ex post penalties at a lower cost simply by reducing α appropriately. Rating inflation in our model therefore takes the form of the CRA setting a low screening intensity, rather than it directly misreporting its signal. A higher fee for a high rating results in more rating inflation and a greater proportion of incorrect ratings. As we note in Appendix B, the equilibrium effect on σ^* cannot be unambiguously signed. Numerically, we find that the price of a high-rated security decreases, and in many examples σ^* increases in f when f is low and decreases in f when f is high.

4 Rating Fee Set by Bargaining

Up to this point, we have taken the rating fee f as exogenously fixed. However, in practice, we expect that the fee represents the outcome of some bargaining process in which the issuer and CRA divide the surplus that is generated in their transaction. In this section, we model the fee as the outcome of a Nash bargaining game between the CRA and issuer. We assume that the bargaining takes place (and the fee is set), before the issuer learns its type. The timing captures the intuition that the screening intensity of the CRA is based on a standard process it applies to all issuers. It also sidesteps the problematic effects of asymmetric information on the analysis of bargaining.N(

Let $\phi \in [0, 1]$ denote the bargaining power of the CRA, with $1 - \phi$ being the bargaining power of the issuer. The disagreement payoff for each party, issuer and CRA, is zero. Therefore, the surplus of each party is their payoff if they reach an agreement.The CRA's expected payoff if an agreement is reached, Ψ, is given by the expression in (1):

$$\Psi = N(\sigma^*)\Big\{\nu(\sigma^*)f + [1 - \nu(\sigma^*)](1 - \alpha^*)(f - \lambda) - kc(\alpha^*)\Big\}, \tag{7}$$

where $N(\sigma^*) = \eta + (1 - \eta)\sigma^*$ is the mass of issuers seeking a rating in equilibrium.

In determining its expected payoff at this stage, the issuer takes into account that if its type is low, it will apply for a rating only with some probability σ^*. The issuer's expected payoff when agreement is reached is

$$\Pi = N(\sigma^*)\Big\{\nu(\sigma^*)\Big(p(\alpha^*,\sigma^*) - f\Big) + (1-\nu(\sigma^*))\Big((1-\alpha^*)[p(\alpha^*,\sigma^*) - f] - m\Big)\Big\}. \quad (8)$$

Note that Ψ and Π are both functions of the equilibrium values of α and σ. That is, the issuer and CRA (correctly) anticipate the equilibrium that will follow in the screening game when they calculate their payoffs from reaching agreement in the bargaining game.

Let $H(f) = \Pi^{1-\phi}\ \Psi^{\phi}$ denote the Nash product. The rating fee f is then the solution to the following problem:

$$\max_f \quad H(f) = \Pi^{1-\phi}\ \Psi^{\phi}. \quad (9)$$

The choice of f affects both the equilibrium values α^* and σ^*, and thus the payoffs Π and Ψ.

The following lemma establishes some characteristics of local optima of the Nash bargaining maximization problem.

Lemma 4. *(a) The Nash product has a local optimum at* $f = \max\{\phi, 1-m\}$.

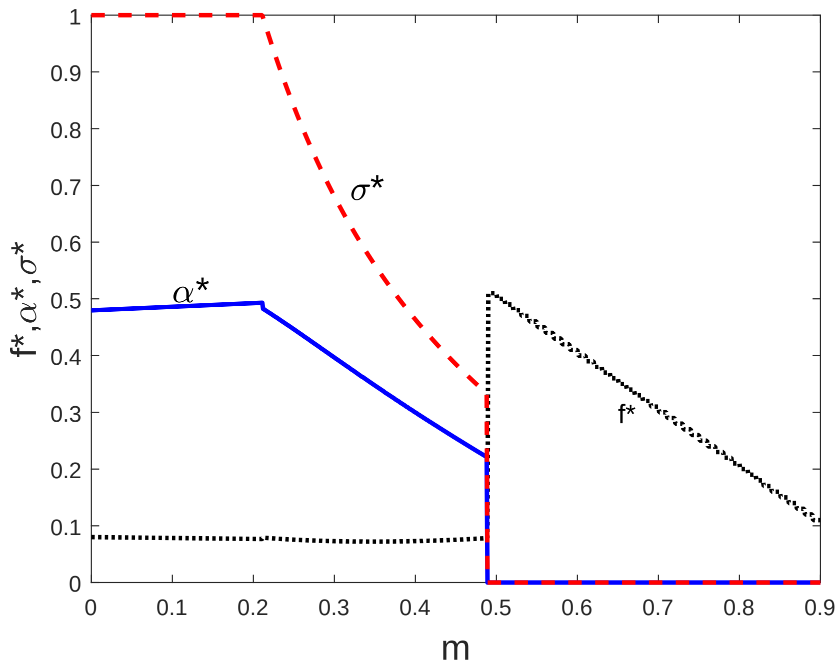
(b) In addition, if m *is sufficiently low, the Nash product has a local optimum in the region* $f \in (0,\lambda)$*. Here, the optimal value of* f *satisfies the equation* $\phi\Pi\frac{d\Psi}{df} = -(1-\phi)\Psi\frac{d\Pi}{df}$.

Recall from Proposition 1 that when $f = 1-m$, the equilibrium in the continuation game features $\sigma^* = \alpha^* = 0$. Part (a) of the Lemma says that when $m \geq 1-\phi$, a local optimum is attained at $f = \phi$. In fact, in this case, the value $f = \phi$ represents a global optimum. When $m < 1-\phi$, a local optimum is attained at $f = 1-m$. In this case, when the manipulation cost m is sufficiently high, the local optimum at $f = 1-m$ is unique. However, for lower values of m, there is a second local optimum, at an interior value of f between 0 and λ.

The overall optimal value of f is determined as follows. For sufficiently large values of m, we have $f^* = \max\{\phi, 1-m\}$. In this case, the equilibrium of the overall game features zero manipulation. For lower values of m, the optimal value f^* lies between 0 and λ. Here, for low values of m, the equilibrium of the overall game features full manipulation, with $\sigma^* = 1$. For intermediate values of m, the equilibrium features partial manipulation, with $\sigma^* \in (0,1)$. In both cases, the equilibrium screening intensity α^* satisfies the CRA's first order condition as shown in Lemma 1.

We illustrate how f^* varies with the manipulation cost using an example. Set $\eta = 0.6, \lambda = 0.2, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$. With the quadratic cost function, although $c'(\alpha) = 2\alpha$

stays bounded even when $\alpha \to 1$, we ensure numerically in this example and in the other examples below that α^* stays strictly below 1.



This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as the manipulation cost m changes. The other parameters are $\eta = 0.6, \lambda = 0.2, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 1: Equilibrium values of f, α, and σ as manipulation cost m changes

Figure 1 exhibits the equilibrium values of f, σ, and α. When $m \leq 0.21$ (approximately), the equilibrium features full-manipulation, with $\sigma^* = 1$, with $f^* \approx 0.08$. For m approximately between 0.21 and 0.48, the equilibrium features partial manipulation, with $\sigma^* \in (0, 1)$. Here, the equilibrium fee f^* remains approximately between 0.07 and 0.1, and less than the value of λ (0.2). At these values of m, there remains a second local optimum for the fee at $f = 1 - m$. As m increases beyond approximately 0.48, the second local optimum becomes the global optimum, so that in this region we have $f^* = 1 - m$, and the equilibrium features zero manipulation.

While the specific thresholds are different, qualitatively the nature of the equilibrium depends on the manipulation cost m in the same way it does when f is taken as exogenous (see Proposition 1). That is, the low-type issuer always seeks a rating when the manipulation cost is low, mixes between seeking a rating and not at intermediate values of the manipulation cost, and stays out of the market altogether at high manipulation costs (above $1 - \phi$).

Observe that in all equilibria of the overall game, the fee f^* is strictly less than the price p. If there is no manipulation, then $p = 1$. Further, $\nu(0) = 1$, so that $\alpha^* = 0$. Thus, the equilibrium fee is $\phi p < p$. Further, in any equilibrium with manipulation (partial or full), it must be that $(1 - \alpha^*)(p - f^*) \geq m$, which implies that $f^* \leq p - \frac{m}{1-\alpha^*} < p$. Thus, when f^* is endogenously chosen, in all equilibria the high-type issuer requests a rating with probability one.

We do not revisit all of the comparative statics from Section 3. However, we do consider two analytic comparative static results. The first comparative static we consider relates to the effect of a changing manipulation cost m in a full-manipulation equilibrium. As we comment on page 16, when f is fixed, a small change in m has no effect in a full-manipulation equilibrium, provided the low-type issuer strictly prefers to manipulate. In contrast, we show that when f is endogenous, a small increase in m increases the equilibrium screening intensity of the CRA, α^*.

Proposition 4. *Suppose that the fee f is endogenously determined and m is low enough that a full-manipulation equilibrium obtains. Then, $\frac{d\sigma^*}{dm} = 0$, but $\frac{d\alpha^*}{dm} > 0$. That is, the CRA's screening intensity increases with m. Further, the price of a high-rated security, p, increases with m, and the equilibrium rating fee, f^*, decreases with m.*

While an increase in m does not affect σ^* in a full-manipulation equilibrium, it decreases the issuer's payoff. As the parties anticipate a decreased payoff to the issuer from reaching an agreement, the bargaining at the initial stage results in a lower equilibrium rating fee f^*. The net cost to the CRA of erroneously assigning a high rating to a low-quality asset is $\lambda - f^*$. A decrease in f^* increases this net cost. As a result, the CRA screens more intensely. Finally, the fact that σ^* is unchanged whereas α^* has increased immediately implies that the price of the high-rated security, p, increases.

This result, which may be seen in Figure 1, demonstrates the potential importance of accounting for the effects of changes in exogenous parameters on the bargaining outcome: with an exogenous rating fee, the CRA's strategy does not change with m. It also demonstrates that imposing penalties on the issuer can, in some circumstances, improve the overall quality of the screening process and reduce the incidence of ratings errors by lowering the benefit to the CRA of making such errors.

Our second comparative statics result considers changes in η and augments Proposition 2.

Proposition 5. *When the fee f is endogenously determined,*

(i) In a partial-manipulation equilibrium, $\frac{d\sigma^*}{d\eta} > 0$, $\frac{d\alpha^*}{d\eta} = 0$, and $\frac{dp(\alpha^*,\sigma^*)}{d\eta} = 0$, as in Proposition 2 when f is taken as fixed. In addition, $\frac{df^*}{d\eta} = 0$. That is, small changes in η, the ex ante probability of the high type, have no effect on the equilibrium fee.

(ii) In a full-manipulation equilibrium, small changes in η lead to a decrease in α^*, an increase in p, and an increase in f^*.

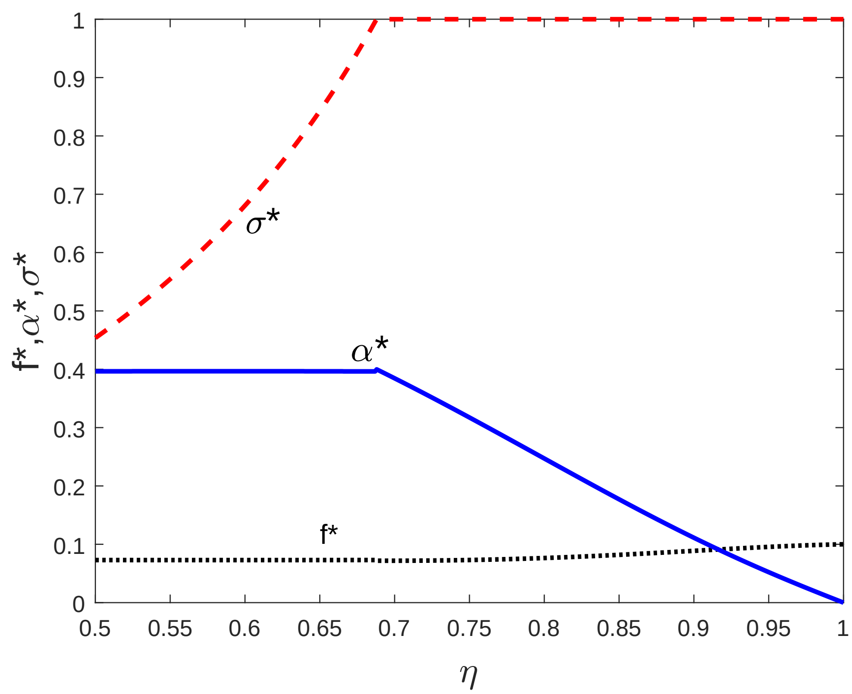
We illustrate this comparative static in the context of an example. Set $\lambda = 0.2, m = 0.3, \phi = 0.1, k = 0.05$ and $c(\alpha) = \alpha^2$.

As seen in Figure 2, when $\eta < \frac{1}{\sqrt{2}} \approx 0.707$, the equilibrium features partial manipulation by the low type issuer, with $\sigma^* \in (0,1)$. In this region, σ^* increases with η, whereas α^* is flat in η. At first glance, the latter property is puzzling. As the expected quality of the issuer improves, one may expect that the need for screening declines. However, the endogenous response of the low-type issuer implies that the average quality of the pool that applies for a rating, in fact, remains constant as η increases in this range. In contrast, when $\eta > 0.707$, the equilibrium features full manipulation, with $\sigma^* = 1$. In this region, as expected, α^* decreases with η.

4.1 Comparative Statics in k, λ, and ϕ

We illustrate through examples how the equilibrium strategies of the low-type issuer (σ^*) and the CRA (α^*) vary with the exogenous parameters k, λ, and ϕ when f is determined endogenously. In these examples, we work with the base set of parameters $\eta = 0.6$, $\lambda = 0.2$, $m = 0.3$, $\phi = 0.1$, $k = 0.05$, and $c(\alpha) = \alpha^2$, unless otherwise noted. In each example, we vary one of the parameters and plot α^* and σ^* as functions of that specific parameter. Figure 3 presents plots where we vary first (a) k and then (b) λ.

These figures provide a contrast to the results in Proposition 3. In Figure 3 (a), σ^* increases in k until k is approximately equal to 0.05, and then flattens out over the range $k \in (0.05, 0.065)$. For k above 0.065, the optimal value of f jumps discretely to $1-m = 0.7$. At this point, therefore, σ^* falls to zero and stays there for higher levels of k. From Proposition



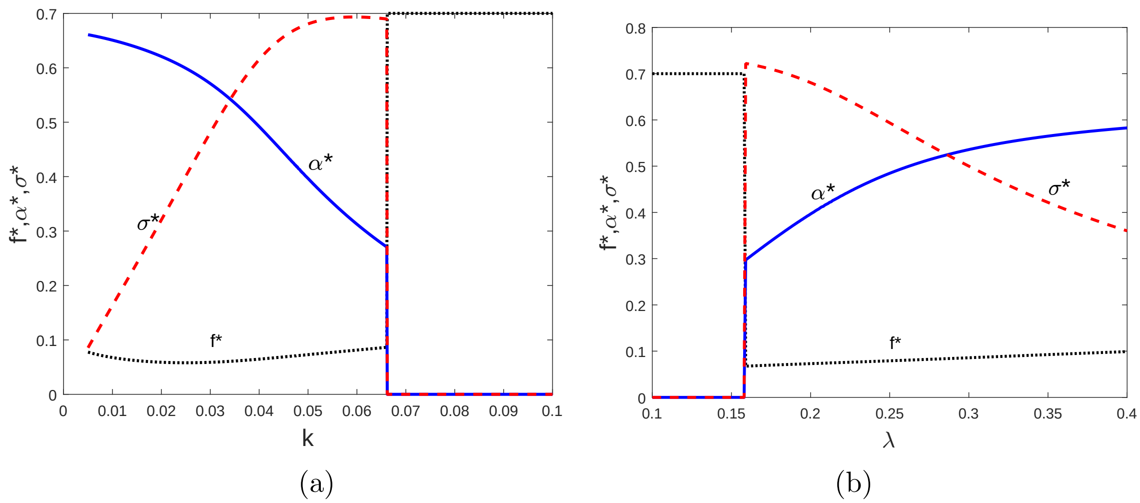
This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as η, the prior probability of a high-type issuer, changes. The other parameters are $\lambda = 0.2, m = 0.3, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 2: Equilibrium values of f, α, and σ as η, the prior probability of a high-type issuer, changes

3, if f were held constant, we would expect to see a continuous decrease in σ^* as k increased above 0.065.

A similar contrast occurs in Figure 3 (b), when λ varies. For low values of λ (below about 0.16), the optimal value of f is at $1 - m = 0.7$, so that we have zero manipulation. As λ increases above 0.16, f^* falls discretely to about 0.08. From this point on, σ^* decreases in λ.

Finally, Figure 4 presents a plot where we vary ϕ, again setting the other parameter values to those noted at the beginning of this section. An increase in ϕ increases the bargaining power of the CRA, which results in a higher rating fee f^*. As the fee for a high rating increases, the CRA's incentives to screen decline, resulting in a lower α^*. Recall that throughout, there remains a local optimum for f at $1 - m$ (in the example, $1 - m = 0.7$ remains well greater than the values of ϕ we consider). At ϕ approximately equal to 0.135, the equilibrium value of f jumps discretely to $1 - m$, with σ^* and α^* both equal to zero at this point.



These figures illustrate the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as k and λ vary. The other parameters are $\eta = 0.6, m = 0.3, \phi = 0.1$, and $c(\alpha) = \alpha^2$. For figure (a), $\lambda = 0.2$, and for figure (b), $k = 0.05$

Figure 3: Effect on α^* and σ^* as k and λ vary

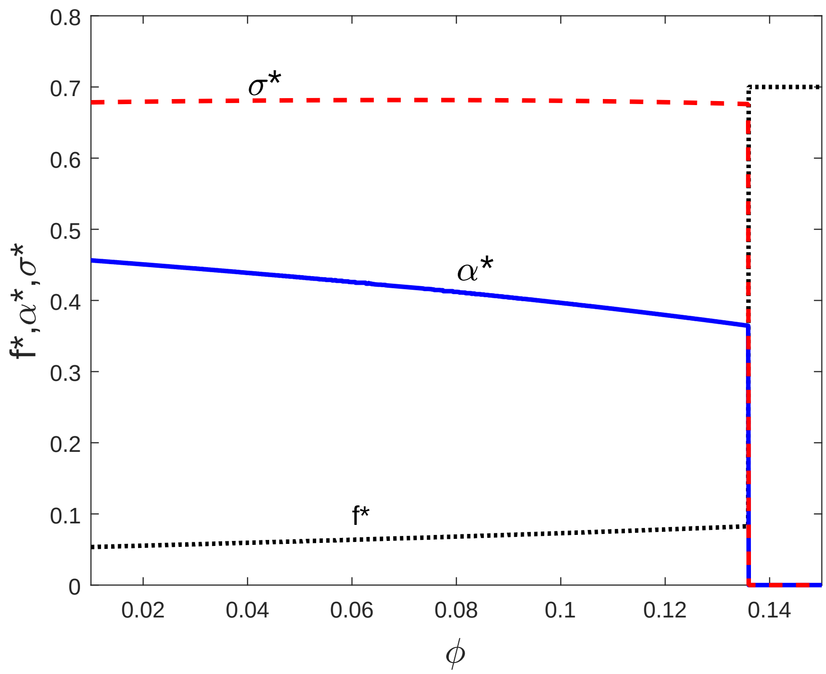
5 Implications

This section discusses some implications of our model. These implications follow primarily from the comparative statics of the model in which the fee f is endogenously determined. We focus primarily on factors driving CRA screening intensity and the incidence of issuer manipulation. At least some of the comparative statics are somewhat counter-intuitive, and so are distinctive to our model, potentially allowing for sharp tests.

Screening intensity (α in the model) can be measured in several different ways: by the number of analysts working for a CRA per security rated, the CRA's expenses per security rated, and the length and detail of reports accompanying ratings. It is important to note that one cannot use the ex-post accuracy of ratings directly as a measure of screening intensity—accuracy in the model is a function of both the CRA's screening intensity, α, and the low-type issuer's manipulation probability, σ.

Higher σ in the model is naturally interpreted as a greater incidence of issuer manipulation. Manipulation is unobservable in real time in our model by assumption. However, ex-post prosecutions or regulatory actions for fraudulent reporting might be a useful indicator of the frequency with which such manipulation occurs.

One implication of our model (Proposition 5) is that CRA screening intensity may be invariant to improvements in the *a priori* quality of assets in a class, especially when that



This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as ϕ, the bargaining power of the CRA, changes. The other parameters are $\eta = 0.6, \lambda = 0.2, m = 0.3, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 4: Equilibrium values of f, α, and σ as ϕ, the bargaining power of the CRA, varies

quality starts from a fairly low level. The parameter η captures information about asset class quality in the model. One could imagine, in practice, capturing asset quality at a broad level through measures of the state of the economy such as GDP or the aggregate market-wide Tobin's Q. One could also imagine measuring asset class-specific quality through current default rates in the asset class.

A second implication of our model (Proposition 4) is that CRA screening intensity may increase with the cost of manipulation, especially when that cost starts from a fairly low level. The parameter m in the model captures the cost of manipulating the CRA's information. One source of variation in manipulation cost is the novelty of the security. The cost of manipulation is likely to be lower for newer securities, as the CRA (and the rest of the market) will learn about the features of a security over time. Another source of variation is the complexity of a security. Manipulation is easier (and hence less costly) when a security is more complex. Expected manipulation costs should also increase with the ex post legal sanctions for committing fraud, the prosecutorial resources available to the regulator, and the willingness of courts to convict in fraud cases. For example, the 2002 Sarbanes-Oxley Act can be interpreted as a large positive shock to the cost of manipulating corporate debt ratings, as manipulation of accounting information is an important means of influencing CRA beliefs.[14]

Conversely, the 2010 Dodd-Frank Act can be interpreted to have reduced the cost of manipulation, in the sense of allowing issuers to hide more information from a CRA. Prior to the act, CRA's were exempt from Regulation Full Disclosure or Reg FD, which prohibited selective disclosure of information to some market participants. The Act extended Reg FD to cover CRAs as well, so that any information disclosed by an issuer to a CRA must also be disclosed to investors at large.

A third implication of our model (Figure 3) is that the incidence of issuer manipulation may decline or stay approximately constant with the cost of screening (k in the model) and increase with the cost to the CRA of ratings errors (λ in the model). One might imagine capturing information about the cost of screening through measures of the opacity of the assets being rated. All else equal, more opaque assets are likely to be more difficult to evaluate, necessitating greater CRA effort to screen out lower-quality assets. One might imagine using information about the severity of any sanctions CRAs face for inaccurate ratings, the likelihood of investor lawsuits, or the importance of the CRA's reputation to measure the overall cost to the CRA of ratings errors.

Alternatively, an increase in competition among CRAs is consistent with there being a lower value to maintaining a reputation, and thus lower penalties for failing to maintain a reputation. Along these lines, Becker and Milbourn (2011) find that ratings become more favorable and less accurate in response to an increase in competition due to the material entry of Fitch as a third major rating agency.

A fourth implication (Figure 4) of our model is that an increase in CRA bargaining power over fees in the issuer-pays model generally leads to less intense CRA screening but has ambiguous implications for the incidence of manipulation. Specifically, when the CRA has little bargaining power, an increase in that bargaining power causes an increase in the incidence of manipulation. However, when the CRA already has a strong bargaining position, a further increase in its bargaining power causes a decrease in the incidence of manipulation. One could measure ϕ directly by observing the level of CRA fees and profitability. In addition, variation in competition among CRAs over time or across markets would also provide a source

[14] As Begley (2015) shows, some firms even incur real costs (such as reducing R&D expenditures) to reduce their debt-to-EBITDA ratio in the year before issuing a new security, in an attempt to obtain a more favorable credit rating.

of variation in CRA bargaining power vis-à-vis issuers and hence their share of proceeds.

Finally, we highlight one policy implication of the model. An important goal of regulatory policy is to minimize rating errors. Our model highlights that the quality of the rating process depends both on the behavior of the CRA and the behavior of the issuer. The model implies that policies that increase the cost of manipulation to the issuer (such as, e.g., stricter disclosure requirements) can both directly increase the accuracy of ratings and also have the side-effect of improving screening by the CRA. Conversely, if a security has a low manipulation cost and increasing this cost is not feasible, strengthening the penalty on the CRA for rating errors leads to a better rating process.

6 Conclusion

We argue that strategic disclosure by issuers is an important friction to consider in the ratings process. Our broad message is that the quality of credit ratings depends on both the quality of screening and the type and disclosure strategy of the issuer. In our model, an endogenous increase in CRA screening intensity may be accompanied by either a decrease or increase in issuer manipulation in response. A decrease in manipulation magnifies the effect of increased screening intensity on ratings accuracy, while an increase in manipulation dampens the effect. In addition, the need to account for issuer behavior can undo the effects of shocks that might otherwise impact the quality of CRA screening.

When assessing periods during which the quality of ratings has been perceived to be low, it is important to remember that issuers are likely to know more about their own asset qualities than a CRA. Any policy design intended to improve the overall ratings process must include providing incentives to issuers to truthfully report the quality of their assets as an important component.

Appendix A: Proofs

Proof of Lemma 1. As mentioned in the text, the payoff of the CRA is $\Psi = N(\tilde{\sigma})\Big\{\nu f + (1-\nu)(1-\alpha)(f-\lambda) - kc(\alpha)\Big\}$. Holding f fixed, the derivative with respect to α is $\frac{\partial \Psi}{\partial \alpha} = (1-\nu)(\lambda - f) - kc'(\alpha)$. If $\lambda \geq f$, the first-order condition implies that the CRA's optimal choice of α satisfies $c'(\alpha) = \frac{1-\nu}{k}(\lambda - f)$. As $c''(\alpha) > 0$, it follows immediately that the second-order condition for a maximum is satisfied.

If $\lambda < f$, we have $\frac{\partial \Psi}{\partial \alpha} < 0$. The optimal choice of α is therefore zero. □

Proof of Lemma 2. Suppose the low-type issuer seeks a rating. Seeking a rating and not manipulating is equivalent to not seeking a rating (since the issuer always gets a low rating, and is therefore revealed to be a low type). If the issuer seeks a rating and manipulates its information, it obtains an expected payoff of $(1-\tilde{\alpha})(\tilde{p} - f) - m$. If it does not seek a rating, it does not issue a financial claim and obtains zero. The statement of the lemma follows immediately. □

Proof of Proposition 1. (i) Suppose that $m \leq (1-\alpha_1)(p(\alpha_1, 1) - f)$, where α_1 is defined as $\alpha_1 = c'^{-1}\left(\frac{1-\eta}{k}\max\{\lambda - f, 0\}\right)$. If the CRA chooses $\alpha^* = \alpha_1$ and the low-type issuer chooses $\sigma^* = 1$ (i.e., seeks a rating with probability one), the price of the high-rated asset is $p(\alpha_1, 1)$. From Lemma 2, it follows that if the CRA chooses $\alpha^* = \alpha_1$, it is a best response for the low-type issuer to set $\sigma^* = 1$. When $\sigma^* = 1$, $\nu(\sigma) = \eta$, so from Lemma 1, it then follows that it is a best response for the CRA to set $\alpha^* = \alpha_1$. Thus, there is an equilibrium in which $(\alpha^*, \sigma^*) = (\alpha_1, 1)$. Further, α_1 is a best response only if $\sigma^* = 1$, and $\sigma = 1$ is a best response only if $m \leq (1-\alpha^*)(p(\alpha^*, 1) - f)$. Therefore, in equilibrium, we can have $\alpha^* = \alpha_1$ only if $m \leq (1-\alpha_1)p(\alpha_1, 1) - f$.

(ii) Suppose that $m \geq 1 - f$. If the CRA chooses $\alpha^* = 0$ and the low-type issuer chooses $\sigma^* = 0$, the price of the high-rated security is 1. Now, from Lemma 2, it is a best response for the low-type issuer to not seek a rating if $m \geq 1 - f$. Further, when $\sigma^* = 0$, $\nu(\sigma^*) = 1$, so that from Lemma 1, the best response of the CRA is to set $\alpha^* = 0$. Therefore, there is an equilibrium in which $(\alpha^*, \sigma^*) = (0, 0)$. Further, an equilibrium with $\alpha^* = 0$ can exist only when $m \geq 1 - f$. At any lower value of m, $\sigma^* > 0$ when $\alpha^* = 0$, so that $\alpha^* = 0$ cannot be a best response.

(iii) Suppose that $m \in ((1-\alpha_1)(p(\alpha_1,1)-f), 1-f)$ and consider the conjectured equilibrium given by $\alpha = c'^{-1}\left(\frac{1-\nu(\sigma)}{k}\max\{\lambda - f, 0\}\right)$ and $\sigma = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha}}{(1-\alpha)(1+f+\frac{m}{1-\alpha})}\right)$. The price of the high-rated security is then $p(\alpha,\sigma) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$. From Lemma 1, it is immediate that α is a best response to σ. Suppose that the low-type issuer is indifferent between seeking a rating and not seeking a rating. Then, it must be that $(1-\alpha)(p(\alpha,\sigma)-f) = m$, or that $(1-\alpha)\left(\frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)} - f\right) = m$. Solving the last equation for σ yields the value of σ^* in the statement of the proposition. It is immediate that $\sigma^* > 0$. Further, $\sigma^* \geq 1$ implies that $m \leq (1-\alpha_1)(p(\alpha_1,1)-f$. Thus, when $m > (1-\alpha_1)(p(\alpha_1,1)-f)$, we have $\sigma^* < 1$. To complete the proof, note that given σ^*, the value of α^* must satisfy the CRA's first-order condition. □

Proof of Lemma 3. Suppose that $f < \lambda$, and let (α^*, σ^*) denote the equilibrium values in a partial-manipulation equilibrium. Observe that $\nu(\sigma^*) = \left(1+\left(\frac{1-\eta}{\eta}\right)\sigma^*\right)^{-1} = \left(1+\frac{1-f-\frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right)^{-1}$, where the last equation uses the expression for σ^* in equation (4). Substituting this expression for $\nu(\sigma^*)$ into the RHS of equation (5) and re-writing the resulting equation, we have that α^* satisfies the condition $h(\alpha^*) = 0$, where

$$h(\alpha) = \left(2-\alpha\left(1+f+\frac{m}{1-\alpha}\right)\right)c'(\alpha) - \frac{\lambda-f}{k}\left(1-f-\frac{m}{1-\alpha}\right). \qquad (10)$$

Observe that $h(0) < 0$. Thus, a sufficient condition for uniqueness is that whenever $h(\alpha) = 0$, we also have $h'(\alpha) > 0$. This condition is equivalent to

$$\left(2-\alpha\left(1+f+\frac{m}{1-\alpha}\right)\right)c''(\alpha) > \left(1+f+\frac{m}{(1-\alpha)^2}\right)c'(\alpha) - \frac{\lambda-f}{k}\frac{m}{(1-\alpha)^2}. \qquad (11)$$

In a partial-manipulation equilibrium, it must be that $\alpha^* < 1-\frac{m}{1-f}$ (otherwise $\sigma^* = 0$). Thus, it follows that $2-\alpha\left(1+f+\frac{m}{1-\alpha}\right) > \frac{2m}{1-f}$, so a sufficient condition for inequality (11) to hold is that

$$\left(\frac{2m}{1-f}\right)c''(\alpha) > \left(1+f+\frac{m}{(1-\alpha)^2}\right)c'(\alpha) - \frac{\lambda-f}{k}\frac{m}{(1-\alpha)^2}. \qquad (12)$$

To obtain condition (6), observe that the equilibrium α^* satisfies $\frac{\lambda-f}{k} = \frac{c'(\alpha^*)}{1-\nu}$. Hence, inequality (12) can be written as $\left(\frac{2m}{1-f}\right)c''(\alpha) > \left(1+f-\frac{\nu}{1-\nu}\frac{m}{(1-\alpha)^2}\right)c'(\alpha)$. Since $\nu > \eta$ in any partial-manipulation equilibrium and since $\frac{m}{(1-\alpha)^2} > m$, the RHS of this inequality is therefore strictly less than $\left(1+f-\frac{\eta m}{1-\eta}\right)c'(\alpha)$. Therefore, inequality (11) is satisfied if

$$\left(\frac{2m}{1-f}\right)c''(\alpha) > \left(1+f-\frac{\eta m}{1-\eta}\right)c'(\alpha). \tag{13}$$

This condition is equivalent to the one stated in the lemma. □

Proof of Proposition 2. As in the proof of Lemma 3, let

$$h(\alpha) = \left(2-\alpha\left(1+f+\frac{m}{1-\alpha}\right)\right)c'(\alpha) - \frac{\lambda-f}{k}\left(1-f-\frac{m}{1-\alpha}\right). \tag{14}$$

Recall that a partial-manipulation equilibrium is defined by $h(\alpha^*) = 0$. Thus, for any $x \in \{\eta, k, \lambda, m, f\}$, from the Implicit Function Theorem we have $\frac{d\alpha^*}{dx} = -\frac{\partial h/\partial x}{\partial h/\partial \alpha}$.

By inspection, $\frac{\partial h}{\partial \eta} = 0$, so it follows that $\frac{d\alpha^*}{d\eta} = 0$.

To calculate $\frac{d\sigma^*}{d\eta}$, recall from equation (4) in Proposition 1 that

$$\sigma^* = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right). \tag{15}$$

It follows that, for any $x \in \{\eta, k, \lambda, m, f\}$, we have $\frac{d\sigma^*}{dx} = \frac{\partial \sigma^*}{\partial x} + \frac{\partial \sigma^*}{\partial \alpha^*}\frac{d\alpha^*}{dx}$. As $\frac{d\alpha^*}{d\eta} = 0$ and (by inspection) $\frac{\partial \sigma^*}{\partial \eta} > 0$, it is immediate that $\frac{d\sigma^*}{d\eta} > 0$.

Finally, note that in a partial-manipulation equilibrium, it must be that $(1-\alpha^*)(p-f) = m$. For small changes in the exogenous parameters, the equilibrium continues to feature partial manipulation, so this equation must continue to hold. As α^* is invariant to η, it follows that p must also remain constant for small changes in η. □

Proof of Proposition 3. Let $y = \frac{\lambda-f}{k}$ and recall the definition of $h(\alpha)$ from equation (14) in the proof of Proposition 2. The Implicit Function Theorem implies that $\frac{d\alpha^*}{dy} = -\frac{\partial h/\partial y}{\partial h/\partial \alpha}$. Further, the second-order condition for α^* requires that $\frac{\partial h}{\partial \alpha} > 0$. By inspection, $\frac{\partial h}{\partial y} < 0$. Thus, it follows immediately that $\frac{d\alpha^*}{dy} > 0$, which implies that $\frac{d\alpha^*}{d\lambda} > 0$ and $\frac{d\alpha^*}{dk} < 0$.

Next, let

$$g(\alpha) = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha}}{(1-\alpha)(1+f)+m}\right). \tag{16}$$

From equation (4) in Proposition 1, it follows that in a partial-manipulation equilibrium we have $\sigma^* = g(\alpha^*)$. Further, $\frac{d\sigma^*}{dy} = \frac{\partial g}{\partial \alpha}\frac{d\alpha^*}{dy}$ since $\frac{\partial g}{\partial y} = 0$ when $y = \frac{\lambda-f}{k}$. From equation (16), we have

$$\frac{\partial g}{\partial \alpha} = \frac{\eta}{1-\eta}\left(\frac{1-f^2-\frac{2m}{1-\alpha}(1+f)-\frac{m^2}{(1-\alpha)^2}}{[(1-\alpha)(1+f)+m]^2}\right). \tag{17}$$

It is immediate to see that $\frac{\partial g}{\partial \alpha}$ is strictly decreasing in α. Further, $\frac{\partial g}{\partial \alpha}$ is strictly negative at $\alpha = 1 - \frac{m}{1-f}$ (the value of α at which σ^* becomes zero) and strictly positive at $\alpha = 0$ if $(f+m)^2 + 2m < 1$. Thus, if $(f+m)^2+2m < 1$, there exists a threshold $\hat{\alpha} \in (0, 1 - \frac{m}{1-f})$ such that $\frac{d\sigma^*}{dy} > 0$ for $\alpha < \hat{\alpha}$ and $\frac{d\sigma^*}{dy} < 0$ for $\alpha > \hat{\alpha}$. Since α^* increases in y and since y decreases in k, this implies that there exists a $\hat{k}$ such that $\frac{d\sigma^*}{dk} > 0$ for $k < \hat{k}$ and $\frac{d\sigma^*}{dk} < 0$ for $k > \hat{k}$. Similarly, since y increases in λ, there exists a $\hat{\lambda}$ such that $\frac{d\sigma^*}{d\lambda} > 0$ for $\lambda < \hat{\lambda}$ and $\frac{d\sigma^*}{d\lambda} < 0$ for $\lambda > \hat{\lambda}$. □

Proof of Lemma 4. First, consider values of f in the region $(0, \lambda)$. For f close to zero, when $\lambda > 0$, it follows that $\Psi < 0$, so the Nash product is undefined. It is immediate that there must be a minimal level of f, which we denote $\underline{f}$, such that $\Psi \geq 0$ if and only if $f \geq \underline{f}$.

Next, consider values of f just below $1-m$. Observe that in this region, as $f \geq \lambda$, it must be that $\alpha^* = 0$.

For f close to but strictly less than $1-m$, from Proposition 1 it follows that $\sigma^* \in (0,1)$. Then, the indifference condition of the low-type issuer implies that $p - f = m$, or $f = p + m$. Observe that when $\alpha^* = 0$, we can write the ex ante payoff of the issuer as $\Pi = \eta(p-f) + (1-\eta)\sigma(p-f-m) = \eta(p-f) = \eta m$ when $\sigma \in (0,1)$. Thus, here Π is invariant to f.

Now, the payoff of the CRA may be written as $\Psi = \Big(\eta + (1-\eta)\sigma\Big)(p+m) - (1-\eta)\sigma\lambda$. When $\alpha^* = 0$, we have $p = \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}$. Thus, $\Psi = \eta(1+m) - (1-\eta)\sigma(1+\lambda-m)$. When $\lambda < 1-m$, it follows that $m < 1+\lambda$. Therefore, Ψ is strictly decreasing in σ, and hence is maximized at $\sigma = 0$. When $\alpha^* = 0$, to induce $\sigma = 0$, it must be that $f = 1-m$.

Now, consider values of $f > 1-m$. At these values, from Proposition 1, we have $\sigma = 0$ and $\alpha = 0$. Therefore, for f in this region, we have $\Psi = \eta f$, and $\Pi = \eta(1-f)$. From the Nash maximization problem, it now follows that the optimal value of f is $\max\{\phi, 1-m\}$, where $1-m$ is the smallest value of f such that $\alpha = \sigma = 0$. As $\phi < 1-m$ by assumption, the optimal value of f is $1-m$.

Hence, the Nash product has a local optimum at $f = 1-m$. □

Proof of Proposition 4. Suppose we are in a full-manipulation equilibrium, so that $\sigma^* = 1$. Recall that $\nu(1) = \eta$. Consider a small increase in m. For a sufficiently small change, it continues to be the case that $\sigma^* = 1$ in the new equilibrium. However, keeping α fixed, the expected surplus of the issuer falls, as the payoff of the low type decreases in m. Therefore, f^* must fall. Further, the first-order condition of the CRA, given by $kc'(\alpha^*) = (1-\eta)(\lambda - f^*)$, implies that as f^* falls, α^* increases. As α^* increases, and σ^* remains at 1, it is immediate that the price p must increase. $\square$

Proof of Proposition 5. (i) As in the case with a fixed rating fee f, the screening intensity α^* in a partial-manipulation equilibrium has to satisfy the condition $h(\alpha^*) = 0$, where $h(\alpha)$ is defined in equation (14) (with $f = f^*$). In addition, α^* and f^* have to satisfy the equation $\phi\Pi\frac{d\Psi}{df} = -(1-\phi)\Psi\frac{d\Pi}{df}$. Note that $h(\alpha)$ does not depend on η; further, f^* depends on η only through $\nu(\sigma^*)$. Recall from the proof of Lemma 3 that in a partial-manipulation equilibrium $\nu(\sigma^*)$ can be expressed as $\nu(\sigma^*) = \left(1 + \frac{1-f-\frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right)^{-1}$. This means that the effect of a (small) increase in η on ν is offset by an increase in σ^*, leaving ν unchanged. Of course, if ν is unchanged, so are α^*, f^*, and $p(\alpha^*, \sigma^*)$.

(ii) Consider a full-manipulation equilibrium. Since $\bar{m}$ is increasing in η, an increase in η means that we continue to have $\sigma^* = 1$. The equilibrium screening intensity satisfies $kc'(\alpha^*) = (1-\eta)(\lambda - f^*)$. As η increases, keeping σ^* fixed at 1, ν increases. Therefore, α^* decreases. Nevertheless, the increase in η implies that p increases, and in turn f^* increases. $\square$

Appendix B: Additional Comparative Statics with Fixed f

In this section, we consider the comparative statics of α^* and σ^* for the case in which the rating fee f is fixed, as the manipulation cost m and rating fee f vary. Suppose throughout that we have a partial-manipulation equilibrium, and suppose further that $f < \lambda$.

First, consider changes in f. Following the technique used in the proof of Proposition 3, observe that when $y = \frac{\lambda - f}{k}$, it is immediate that $\frac{dy}{dk} < 0$. As $\frac{d\alpha^*}{dy} > 0$ in a partial-manipulation equilibrium, it follows that $\frac{d\alpha^*}{df} < 0$.

Now, as in the proof of Proposition 3, we have $\frac{d\sigma^*}{df} = \frac{\partial g}{\partial f} + \frac{\partial g}{\partial \alpha}\frac{d\alpha^*}{df}$, where $g(\alpha) = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha}}{(1-\alpha)\left(1+f+\frac{m}{1-\alpha}\right)}\right)$. It is immediate that $\frac{\partial g}{\partial f} < 0$. Thus, if $\frac{\partial g}{\partial \alpha} > 0$ (i.e., when $(f+m)^2 + 2m < 1$ and α is small), it follows that $\frac{d\sigma^*}{df} < 0$. However, if α is large or $(f+m)^2 + 2m > 1$, then $\frac{\partial g}{\partial \alpha} < 0$, so it is not immediate to determine the sign of $\frac{d\sigma^*}{df}$.

Next, consider changes in m, the manipulation cost for the low-type issuer. We have $\frac{\partial h}{\partial m} = \frac{-\alpha c'(\alpha) + \frac{\lambda - f}{k}}{1-\alpha}$. Recalling that α^* is defined by the equation $c'(\alpha^*) = \frac{\lambda - f}{k}(1 - \nu(\sigma^*(\alpha^*)))$, we have $\frac{\partial h}{\partial m} = \frac{\lambda - f}{k(1-\alpha)}(-\alpha(1-\nu) + 1) = \frac{\lambda - f}{k(1-\alpha)}(1 - \alpha + \nu\alpha) > 0$. As $\frac{\partial h}{\partial \alpha} > 0$, it follows that $\frac{d\alpha^*}{dm} = -\frac{\partial h/\partial m}{\partial h/\partial \alpha} < 0$.

Now, observe that α^* and σ^* must satisfy the equation $c'(\alpha^*) = \frac{\lambda - f}{k}(1 - \nu(\sigma^*))$. Consider a small increase in m. As α^* decreases, so does $c'(\alpha^*)$. Therefore, it must be that $\nu(\sigma^*)$ increases, which in turn implies that σ^* decreases. Therefore, $\frac{d\sigma^*}{dm} < 0$. ■

References

Becker, B., and T. Milbourn, 2011, "How did increased competition affect credit ratings?," *Journal of Financial Economics*, 101(3), 493–514.

Begley, T., 2015, "The Real Costs of Corporate Credit Ratings," Working Paper, SSRN.

Bolton, P., X. Freixas, and J. Shapiro, 2012, "The Credit Ratings Game," *Journal of Finance*, 67, 85–112.

Bouvard, M., and R. Levy, 2013, "Two-Sided Reputation in Certification Markets," Working Paper, SSRN.

Brandenburger, A., and B. Polak, 1996, "When Managers Cover Their Posteriors: Making the Decisions the Market Wants to See," *RAND Journal of Economics*, 27, 523–541.

Cohn, J. B., and U. Rajan, 2013, "Optimal Corporate Governance in the Presence of an Activist Investor," *Review of Financial Studies*, 26(4), 985–1020.

Cox, G. W., 1994, "A Note on Crime and Punishment," *Public Choice*, 78, 115–124.

Crocker, K. J., and J. Slemrod, 2007, "The Economics of Earnings Manipulation and Managerial Compensation," *RAND Journal of Economics*, 38, 698–713.

Dewatripont, M., I. Jewitt, and J. Tirole, 1999, "The Economics of Career Concerns, Part I: Comparing Information Structures," *Review of Economic Studies*, 66, 183–198.

Frenkel, S., 2015, "Repeated Interaction and Rating Inflation: A Model of Double Reputation," *American Economic Journal: Microeconomics*, 7, 250–280.

Fulghieri, P., G. Strobl, and H. Xia, 2014, "The Economics of Solicited and Unsolicited Credit Ratings," *Review of Financial Studies*, 27, 484–518.

Gill, D., and D. Srgoi, 2012, "The optimal choice of pre-launch reviewer," *Journal of Economic Theory*, 147(3), 1247–1260.

Goel, A. M., and A. V. Thakor, 2015, "Information reliability and welfare: A theory of coarse credit ratings," *Journal of Financial Economics*, 115(3), 541–557.

Goldman, E., and S. L. Slezak, 2006, "An Equilibrium Model of Incentive Contracts in the Presence of Information Manipulation," *Journal of Financial Economics*, 80, 603–626.

Holmström, B., 1999, "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, 66, 169–182.

Khalil, F., 1997, "Auditing without Commitment," *Rand Journal of Economics*, 28(4), 629–640.

Mookherjee, D., and I. Png, 1989, "Optimal Auditing, Insurance, and Redistribution," *Quarterly Journal of Economics*, 104(2), 399–415.

Prat, A., 2005, "The Wrong Kind of Transparency," *American Economic Review*, 95, 862–877.

Prendergast, C., 1993, "A Theory of "Yes Men"," *American Economic Review*, 83, 757–770.

Sangiorgi, F., and C. Spatt, 2015, "Opacity, Credit Rating Shopping and Bias," Working Paper, SSRN.

Skreta, V., and L. Veldkamp, 2009, "Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation," *Journal of Monetary Economics*, 56(5), 678–695.

Song, F., and A. V. Thakor, 2006, "Information control, career concerns, and corporate governance," *The Journal of Finance*, 61(4), 1845–1896.

Stein, J. C., 1989, "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *Quarterly Journal of Economics*, 104(4), 655–669.

Strobl, G., 2013, "Earnings Manipulation and the Cost of Capital," *Journal of Accounting Research*, 51(2), 449–473.

Townsend, R., 1979, "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21, 265–293.

Tsebelis, G., 1990, "Penalty has no impact on crime: A game-theoretic analysis," *Rationality and Society*, 2(3), 255–286.