Factor Momentum and the Momentum Factor*

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Abstract

Momentum in individual stock returns emanates from momentum in factor returns. The returns on most factors are positively autocorrelated: the average factor earns a monthly return of 2 basis points following a year of losses and 52 basis points following a positive year. Factor momentum explains both standard momentum and many other forms of it, such as intermediate, industry, and industry-adjusted momentum. We describe the mechanism by which factor momentum transmits into the cross section of security returns. Momentum crashes when the typically positive autocorrelations in factor returns turn negative. Equity momentum is not a distinct risk factor; it is an aggregation of the autocorrelations present in the factors of the economy.

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1 Introduction

Momentum appears to violate the efficient market hypothesis in its weakest form. Past returns should not predict future returns if asset prices respond to new information immediately and to the right extent—unless past returns correlate with changes in systematic risk. Researchers have sought to explain the profitability of momentum strategies with time-varying risk, behavioral biases, and trading frictions.[1] At the same time, the pervasiveness of momentum over time and across asset classes has given momentum the status of an independent risk factor that explains the cross section of returns.[2] In this paper we show that momentum is not a distinct, standalone risk factor: it owes its existence to the autocorrelation properties of the factors in the economy.

We first show that prior factor returns are informative about factors' future performance. Small stocks, for example, are likely to outperform big stocks when they have done so over the prior year. This effect is economically and statistically large: The average factor earns 52 basis points per month following a year of gains but just 2 basis points following a year of losses. The difference in these average returns is significant with a t-value of 4.86. This result is not specific to the use of obscure asset pricing factors: we work with the major factors that are regularly updated and published by academics and a hedge fund.

A time-series factor momentum strategy is a strategy that bets on this continuation in factor returns. It is long the factors with positive returns and short those with negative returns. This momentum strategy earns an annualized return of 4.2% (t-value = 7.04). We use the Lo and

[1] See, for example, Conrad and Kaul (1998), Berk et al. (1999), Johnson (2002), and Sagi and Seasholes (2007) for risk-based explanations; Daniel et al. (1998), Hong and Stein (1999), Frazzini et al. (2012), Cooper et al. (2004), Griffin et al. (2003), and Asness et al. (2013) for behavioral explanations; and Korajczyk and Sadka (2004), Lesmond et al. (2004), and Avramov et al. (2013) for trading friction-based explanations.

[2] Jegadeesh (1990) and Jegadeesh and Titman (1993) document momentum in the cross section of stocks, Jostova et al. (2013) in corporate bonds, Beyhaghi and Ehsani (2017) in corporate loans, Hendricks et al. (1993), Brown and Goetzmann (1995), Grinblatt et al. (1995), and Carhart (1997) in mutual funds, Baquero et al. (2005), Boyson (2008), and Jagannathan et al. (2010) in hedge funds, Bhojraj and Swaminathan (2006), Asness et al. (2013), and Moskowitz et al. (2012) in major futures contracts, Miffre and Rallis (2007) and Szakmary et al. (2010) in commodity futures, Menkhoff et al. (2012) in currencies, and Lee et al. (2014) in credit default swaps.

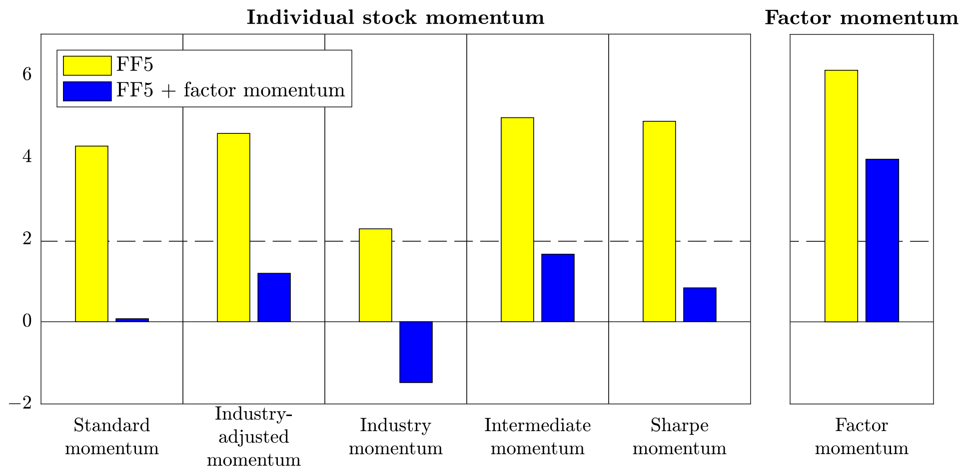

Figure 1: **Individual stock momentum versus factor momentum.** This figure shows t-values associated with alphas for five momentum strategies that trade individual stocks and a factor momentum strategy that trades 20 factors. For individual stock momentum strategies, we report t-values from the five-factor model (yellow bars) and this model augmented with factor momentum (blue bars). For the factor momentum strategy, we report t-values from the five-factor model (yellow bar) and this model augmented with all five individual stock momentum strategies (blue bar). The dashed line denotes t-value = 1.96.

MacKinlay (1990) and Lewellen (2002) decompositions to show that there is an important difference between *cross-sectional* and *time-series* factor momentum strategies. We show that the time-series momentum strategy dominates the cross-sectional strategy because the latter bets on two patterns in factor returns, with one of these bets systematically losing in the data. These two bets are that a factor's high return predicts (1) high returns on the factor itself and (2) low returns on the other factors. It is this second channel that works against the cross-sectional momentum strategy: a high return on a factor predicts high returns on the other factors as well. The time-series momentum strategy dominates the cross-sectional strategy because it is a pure bet on the positive autocorrelation of factor returns.

Momentum in factor returns transmits into the cross section of security returns. The amount of momentum that transmits depend on the amount of cross-sectional variation in factor loadings. The more these loadings differ across assets, the more of the factor momentum shows up as *cross-sectional* momentum in individual security returns. If the autocorrelations in factor returns are the sources of individual stock momentum, factor momentum should subsume individual stock momentum. The data support this prediction. We augment the Fama and French (2015) five-factor model with the time-series factor momentum strategy, and show that this model describes the average returns of portfolios sorted by momentum better than a model that adds the UMD factor of Carhart (1997). That is, a factor that *directly* targets momentum in stock returns does not perform as well as a momentum factor constructed in the space of factor returns.

Factor momentum explains other forms of momentum in addition to the standard momentum of Jegadeesh and Titman (1993): industry momentum, industry-adjusted momentum, intermediate momentum, and Sharpe ratio momentum. The left-hand side of Figure 1 illustrates this result. In this figure we report two pairs of t-values for each version of momentum. The first is that associated with the strategy's five-factor model alpha; the second one is from the model that adds factor momentum to the five-factor model. Factor momentum renders all individual stock momentum strategies statistically insignificant. The right-hand side of the same figure shows that a five-factor model augmented with *all five forms of individual stock momentum* leaves factor momentum with an alpha that is significant with a t-value of 3.96. Individual stock momentum emanates from the momentum in factor returns, not vice versa.

Our results suggest that equity momentum is an accumulation of the autocorrelations in factor returns; it is not a distinct risk factor. A momentum strategy can be viewed as an indirect method of factor timing. An investor who trades momentum in individual securities is indirectly timing factors to benefit from the positive autocorrelations. Our results indicate that a strategy that times factors directly dominates all strategies that time factors only indirectly by trading momentum in individual stock returns.

Because momentum is a bet on the continuation in factor returns, the diversification benefits of mixing equity momentum with other factors are limited. We show that (non-momentum) factors offer diversification benefits only when these factors have performed (and are therefore expected to perform) poorly. The diversification benefits obtained by diversifying momentum with value, following value's underperformance, for example, are the same as those we would obtain by diversifying momentum with cash. When value has performed well, its correlation with momentum is positive; it is only when value has performed poorly that it correlates negatively with momentum. The issue is that when value has lost money, it typically delivers average returns indistinguishable from zero.

We show that momentum crashes and booms, reported and studied in Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), stem from time-varying autocorrelations in factor returns. Momentum "booms" occur when the positive autocorrelations in factor returns intensify and momentum crashes occur when these autocorrelations turn negative. Although the negative-autocorrelation regimes are less frequent, in these regimes the negative autocorrelations permeate the entire cross section of factors. These negative autocorrelations manifest as momentum crashes. A theory of momentum would therefore need to explain, first, why factor returns are typically positively autocorrelated and, second, why most of the autocorrelations sometimes, and abruptly, turn negative at the same time.

Our results suggest that a mispricing mechanism may underlie momentum profits. The strength of factor momentum significantly varies by investor sentiment of Baker and Wurgler (2006). Conditional on low sentiment, factors that earned positive return over the prior year outperform those that lost money by 71 basis points per month (t-value = 4.79). In the high-sentiment environment, this performance gap is just 18 basis points (t-value = 1.32). Factor momentum may stem from asset values *drifting* away from, and later towards, fundamental values. Under this interpretation, factors may, at least in part, be about mispricing (Kozak et al., 2018; Stambaugh et al., 2012)

Our results relate to McLean and Pontiff (2016) and Avramov et al. (2017) who show that anomaly returns predict the cross section of anomaly returns at the one-month and one-year lags. We show that the profits of cross-sectional momentum strategies derive almost entirely from the autocorrelation in factor returns; that time-series factor momentum fully subsumes momentum in individual stock returns (in all its forms); that the characteristics of stock momentum returns change predictably alongside the changes in the autocorrelation of factor returns; and that momentum is not a distinct risk factor—rather, momentum factor aggregates up the autocorrelation patterns in other factors. Because almost all factor returns are autocorrelated, for reasons we do not yet understand, stock price momentum is inevitable.

2 Data

We take the primary factor and portfolio data from three public sources: Kenneth French's, AQR's, and Robert Stambaugh's data libraries.[3] Table 1 lists the factors, start dates, average annualized returns, standard deviations of returns, and t-values associated with the average returns. If the return data on a factor is not provided, we use the portfolio data to compute the factor return. We compute factor return as the average return on the three top deciles minus that on the three bottom deciles, where the top and bottom deciles are defined in the same way as in the original study.

The 15 anomalies that use U.S. data are accruals, betting against beta, cash-flow to price, investment, earnings to price, book-to-market, liquidity, long-term reversals, net share issues, quality minus junk, profitability, residual variance, market value of equity, short-term reversals, and momentum. Except for the liquidity factor of Pástor and Stambaugh (2003), the return data for these factors begin in July 1963; those for the liquidity factor begin in January 1968. The seven global factors are betting against beta, investment, book-to-market, quality minus junk, profitabil-

[3]These data sets are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, https://www.aqr.com/Insights/Datasets, and http://finance.wharton.upenn.edu/~stambaug/.

Table 1: Descriptive statistics

This table reports the start date, the original study, and the average annualized returns, standard deviations, and t-values for 15 U.S. and seven global factors. The universe of stocks for the global factors is the developed markets excluding the U.S. The end date for all factors is December 2015.

		Start	Annual return		
Factor	Original study	date	Mean	SD	t-value
U.S. factors					
Size	Banz (1981)	Jul 1963	3.1%	10.6%	2.11
Value	Rosenberg et al. (1985)	Jul 1963	4.0%	9.9%	2.96
Profitability	Novy-Marx (2013)	Jul 1963	3.0%	7.4%	2.92
Investment	Titman et al. (2004)	Jul 1963	3.6%	7.0%	3.77
Momentum	Jegadeesh and Titman (1993)	Jul 1963	8.5%	14.7%	4.18
Accruals	Sloan (1996)	Jul 1963	2.6%	6.7%	2.81
Betting against beta	Frazzini and Pedersen (2014)	Jul 1963	10.1%	11.2%	6.52
Cash-flow to price	Rosenberg et al. (1985)	Jul 1963	3.3%	10.2%	2.36
Earnings to price	Basu (1983)	Jul 1963	4.0%	10.1%	2.89
Liquidity	Pástor and Stambaugh (2003)	Jan 1968	5.0%	12.1%	2.87
Long-term reversals	Bondt and Thaler (1985)	Jul 1963	3.2%	8.7%	2.70
Net share issues	Loughran and Ritter (1995)	Jul 1963	3.1%	8.2%	2.73
Quality minus junk	Asness et al. (2017)	Jul 1963	4.2%	8.2%	3.73
Residual variance	Ang et al. (2006)	Jul 1963	1.5%	17.6%	0.64
Short-term reversals	Jegadeesh (1990)	Jul 1963	5.9%	10.9%	3.92
Global factors					
Size	Banz (1981)	Jul 1990	0.4%	7.5%	0.27
Value	Rosenberg et al. (1985)	Jul 1990	4.5%	7.5%	3.04
Profitability	Novy-Marx (2013)	Jul 1990	4.5%	4.9%	4.63
Investment	Titman et al. (2004)	Jul 1990	2.3%	6.3%	1.87
Momentum	Jegadeesh and Titman (1993)	Nov 1990	8.8%	12.6%	3.49
Betting against beta	Frazzini and Pedersen (2014)	Jul 1990	10.0%	9.9%	5.11
Quality minus junk	Asness et al. (2017)	Jul 1990	5.1%	7.1%	3.57

ity, market value of equity, and momentum. Except for the momentum factor, the return data for these factors begin in July 1990; those for the momentum factor begin in November 1990. We use monthly factor returns throughout this study.

Table 1 highlights the significant variation in average annualized returns. The global size factor, for example, earns 0.4%, while both the U.S. and global betting against beta factors earn 10.0%.

Factors' volatilities also vary significantly. The global profitability factor has an annualized standard deviation of returns of 4.9%; that of the U.S. momentum factor is 14.7%.

3 Factor momentum

3.1 Factor returns conditional on past returns

Table 2 shows that factor returns are significantly predictable by their own prior returns. We estimate time-series regressions in which the dependent variable is factor f's return in month t, and the explanatory variable is an indicator variable for the factor's performance over the prior one-year period, from month $t-12$ to month $t-1$. This indicator variables takes the value of one if the factor's return is positive, and zero otherwise. We also estimate a pooled regression to measure the extent to which prior returns predict the returns of the average factor.

The intercepts in Table 2 measure the average factor returns earned following a year of underperformance. The slope coefficient represents the difference in returns between up- and down-years. In these regressions all slope coefficients, except that for the U.S. momentum factor, are positive and nine of the estimates are significant at the 5% level. Although all factors' unconditional means are positive (Table 1), the intercepts show that eight anomalies earn a negative average return following a year of underperformance. The first row shows that the amount of predictability in factor premiums is economically and statistically large. The average anomaly earns a monthly return of just 1 basis point (t-value = 0.06) following a year of underperformance. When the anomaly's return over the prior year is positive, this return increases by 51 basis points (t-value = 4.67) to 52 basis points.

Table 2: Average factor returns conditional on their own past returns

The table reports estimates from a univariate regression model in which the dependent variable is a factor's monthly return and the independent variable takes the value of one if the factor's average return over the prior year is positive and zero otherwise, $r_t^f = \alpha + \beta \times \mathbf{1}_{\bar{r}_{-t}^f > 0} + \epsilon_t$. We estimate these regressions using pooled data (first row) and separately for each anomaly (remaining rows). In the pooled regression we cluster the standard errors by time.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.01	0.06	0.52	4.67
		U.S. factors		
Size	−0.15	−0.77	0.70	2.76
Value	0.15	0.78	0.28	1.16
Profitability	0.01	0.08	0.38	2.14
Investment	0.14	1.06	0.24	1.44
Momentum	0.78	2.03	−0.10	−0.23
Accruals	0.10	0.79	0.14	0.88
Betting against beta	−0.16	−0.58	1.29	4.10
Cash-flow to price	0.10	0.52	0.28	1.14
Earnings to price	0.16	0.81	0.27	1.07
Liquidity	0.14	0.58	0.44	1.43
Long-term reversals	−0.18	−1.10	0.71	3.42
Net share issues	0.19	1.27	0.10	0.53
Quality minus junk	−0.04	−0.23	0.61	3.04
Residual variance	−0.52	−1.73	1.19	2.89
Short-term reversals	0.34	1.28	0.20	0.66
		Global factors		
Size	−0.08	−0.42	0.23	0.95
Value	−0.04	−0.18	0.63	2.31
Profitability	0.11	0.58	0.32	1.54
Investment	−0.13	−0.75	0.51	2.34
Momentum	0.52	1.03	0.30	0.55
Betting against beta	0.03	0.08	1.15	2.95
Quality minus junk	0.28	1.18	0.20	0.72

3.2 Average returns of time-series and cross-sectional factor momentum strategies

We measure the economic significance of factor momentum by measuring the profitability of strategies that take long and short positions in factors based on their prior returns. A time-series momentum strategy is long factors with positive returns over the prior one-year period (winners) and short factors with negative returns (losers). A cross-sectional momentum strategy is long factors that earned above-average returns relative to the other factors over the prior one-year period (winners) and short factors with below-average returns (losers). We rebalance both strategies monthly.[4] We exclude the two stock momentum factors, U.S. and global UMD, from the set of factors to avoid inducing a mechanical correlation between factor momentum and individual stock momentum. The two factor momentum strategies therefore trade a maximum of 20 factors.

Table 3 shows the average returns for the time-series and cross-sectional factor momentum strategies as well as an equal-weighted portfolio of all 20 factors. The annualized return on the average factor is 4.2% with a t-value of 7.60. In the cross-sectional strategy, both the winner and loser portfolios have, by definition, the same number of factors. In the time-series strategy, the number of factors in these portfolios varies. For example, if there are five factors with above-zero returns and 15 factors with below-zero returns over the one-year period, then the winner strategy is long five factors and the loser strategy is long the remaining 15 factors. The time-series momentum strategy takes positions in all N factors,

$$r_t^{\text{time-series momentum}} = \frac{1}{N} \sum_{f=1}^{N} \text{sign}(\bar{r}_{-t}^{f}) \times r_t^f, \tag{1}$$

where N starts at 14 in July 1963 and increases to 20 by November 1990 because of the variation in the factors' start dates (Table 1). We report the returns both for the factor momentum strategies as well as for the loser and winner portfolios underneath these strategies. These loser and winner strategies are equal-weighted portfolios of the factors assigned into these portfolios.

Consistent with the results on the persistence in factor returns in Table 2, both winner strategies

[4] In Appendix A.1 we construct alternative strategies in which the formation and holding periods range from one month to two years.

Table 3: Average returns of time-series and cross-sectional factor momentum strategies

This table reports annualized average returns, standard deviations, and Sharpe ratios for different combinations of 20 factors. The equal-weighted portfolio invests in all 20 factors with the same weights. The time-series factor momentum strategy is long factors with positive returns over the prior one-year period (winners) and short factors with negative returns (losers). The cross-sectional momentum strategy is long factors that earned above-average returns relative to other factors over the prior one-year period (winners) and short factors with below-average returns (losers). We rebalance all strategies monthly.

	Annualized return			
Strategy	Mean	SD	t-value	Sharpe ratio
Equal-weighted portfolio	4.21	3.97	7.60	1.06
Time-series factor momentum	4.19	4.27	7.04	0.98
Winners	6.26	4.70	9.54	1.33
Losers	0.28	6.38	0.31	0.04
Cross-sectional factor momentum	2.78	3.88	5.14	0.72
Winners	7.15	5.69	9.00	1.26
Losers	1.30	5.33	1.74	0.24

outperform the equal-weighted benchmark, and the loser strategies underperform it. The portfolio of time-series winners earns an average return of 6.3% with a t-value of 9.54, and that of cross-sectional winners has an average return of 7.2% with a t-value of 9.00. The two loser portfolios earn average returns of 0.3% and 1.3%, and the t-values associated with these averages are only 0.31 and 1.74.

The momentum strategies are about the spreads between the winner and loser portfolios. The time-series factor momentum strategy earns an annualized return of 4.2% (t-value = 7.04); the cross-sectional strategy earns a return of 2.8% (t-value = 5.14). Because time-series losers earn premiums that are close to zero, the choice of being long or short a factor following periods of negative returns is inconsequential from the viewpoint of average returns. However, by diversifying across all 20 factors, the time-series momentum strategy has a lower standard deviation than the winner portfolio alone (4.3% versus 4.7%).

Figure 2 plots the cumulative returns associated the equal-weighted portfolio and the winner and loser portfolios of Table 3. We standardize each strategy in this figure so that their volatilities

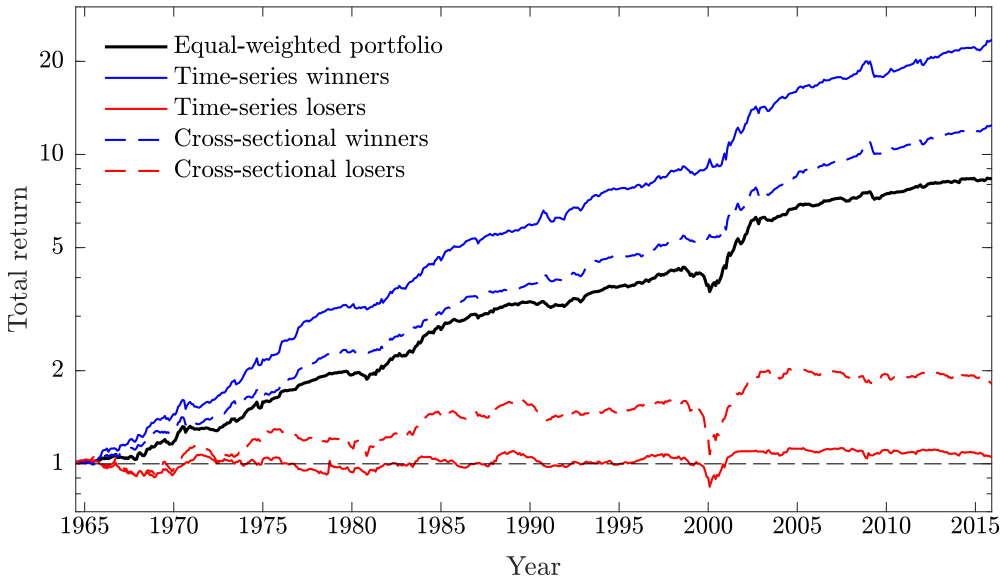

Figure 2: **Profitability of time-series and cross-sectional factor momentum strategies, July 1963–December 2015.** This figure displays total return on an equal-weighted portfolio of all factors and the returns on factors partitioned into winners and losers by their past performance. Time-series winners and losers are factors with above- or below-zero return over the prior one-year period. Cross-sectional winners and losers are factors that have out- or underperformed the average factor over this formation period. Each portfolio is rebalanced monthly and each portfolio's standard deviation is standardized to equal to that of the equal-weighted portfolio.

are equal to that of the equal-weighted portfolio. Consistent with its near zero monthly premium, the total return on the time-series loser strategy remains close to zero even at the end of the 52-year sample period. The time-series winner strategy, by contrast, has earned three times as much as the passive strategy by the end of the sample period. Although the cross-sectional winner strategy in Panel A of Table 3 earns the highest average return, it is more volatility, and so its performance falls short of that of the time-series winner strategy on a volatility-adjusted basis. The cross-sectional loser strategy earns a higher return than the time-series loser strategy: factors that underperformed other factors but that still earned *positive* returns tend to earn high returns the next month. The winner-minus-loser gap is therefore considerably wider for the time-series strategies than what it is for the cross-sectional strategies.

3.3 Decomposing factor momentum profits: Why does the cross-sectional strategy underperform the time-series strategy?

We use the Lo and MacKinlay (1990) and Lewellen (2002) decompositions to measure the sources of profits to cross-sectional and time-series factor momentum strategies. The cross-sectional decomposition chooses portfolio weights that are proportional to demeaned past returns so that the weight is positive for factors with above-average past returns and negative for those with below-average past returns.[5] The weight on factor f in month t is,

$$w_t^f = r_{-t}^f - \bar{r}_{-t},$$

where r_{-t}^f is factor f's past return over some formation period such as from month $t-12$ to month $t-1$ and $\bar{r}_{-t}$ is the cross-sectional average of all factors' returns over the same formation period. The month-t return that results from the position in factor f is therefore

$$\pi_t^f = (r_{-t}^f - \bar{r}_{-t})\, r_t^f, \tag{2}$$

where r_t^f is factor f's return in month t. We can decompose the profits by averaging the profits in equation (2) across the F factors and taking expectations:

$$\mathrm{E}[\pi_t^{\mathrm{XS}}] = \mathrm{E}\big[\sum_{f=1}^{F} \frac{1}{F}(r_{-t}^f - \bar{r}_{-t}) r_t^f\big] = \frac{1}{F}\sum_{f=1}^{F} \mathrm{cov}(r_{-t}^f, r_t^f) - \mathrm{cov}(\bar{r}_{-t}, \bar{r}_t) + \frac{1}{F}\sum_{f=1}^{F}(\mu^f - \bar{\mu})^2, \tag{3}$$

[5] The key idea of the Lo and MacKinlay (1990) decomposition is the observation that, by creating a strategy with weights proportional to past returns, the strategy's expected return is the expected product of lagged and future returns. This expected product can then be expressed as the product of expectations plus the covariance of returns.

where μ^f the unconditional expected return of factor f. The three potential sources of profits can be isolated by writing equation (3) in matrix notation (Lo and MacKinlay, 1990):

$$\begin{aligned} \mathrm{E}[\pi_t^{\mathrm{XS}}] &= \frac{1}{F}\mathrm{Tr}(\Omega) - \frac{1}{F^2}1'\Omega 1 + \sigma_\mu^2 \\ &= \frac{F-1}{F^2}\mathrm{Tr}(\Omega) - \frac{1}{F^2}(1'\Omega 1 - \mathrm{Tr}(\Omega)) + \sigma_\mu^2, \end{aligned} \tag{4}$$

where $\Omega = \mathrm{E}\big[(r_{-t}^f - \mu)(r_t^f - \mu)'\big]$ is the autocovariance matrix of factor returns, $\mathrm{Tr}(\Omega)$ is the trace of this matrix, and σ_μ^2 is the cross-sectional variance of mean factor returns.

The representation in equation (4) separates cross-sectional momentum profits to three sources:

1. Autocorrelation in factor returns: a past high factor return signals future high return,
2. Negative cross-covariances: a past high factor return signals low returns on other factors, and
3. Cross-sectional variance of mean returns: some factors earn persistently high or low average returns.

The last term is independent of the autocovariance matrix; that is, factor "momentum" can emerge even in the absence of any time-series predictability (Conrad and Kaul, 1998). A cross-sectional strategy is long the factors with the highest past returns and short the factors with the lowest past returns; therefore, if past returns provide good measures of factors' unconditional means, a cross-sectional momentum strategy earns positive returns even in the absence of auto- and cross-covariance patterns.

Table 4 shows that, over the 1964–2015 sample period, the linear cross-sectional strategy in equation (4) earns an average annualized return of 2.48% with a t-value of 3.49. The autocovariance term contributes an average of 2.86%, more than all of the cross-sectional strategy's profits. The cross-covariance term is positive and, therefore, it negatively contributes negatively (−1.00% per

Table 4: Decomposition of factor momentum profits

Panel A reports the amount that each term in equation (4) contributes to the profits of the cross-sectional factor momentum strategy. Panel B reports the contributions of the terms in equation (5) to the profits of the time-series factor momentum strategies. We multiply the cross-covariance term by −1 so that these terms represent their net contributions to the returns of the cross-sectional and time-series strategies. The premiums are reported in percentages per year.

Strategy	Annualized Premium (%)	t-value
Cross-sectional factor momentum	2.48	3.49
Autocovariance	2.86	2.96
(−1)× Cross-covariance	−1.00	1.85
Variance of mean returns	0.53	3.41
Time-series factor momentum	4.88	4.65
Autocovariance	3.01	2.96
Mean squared return	1.88	4.41

year) to this cross-sectional strategy's profits. That is, a positive return on a factor predicts positive returns also on the other factors, and the cross-sectional strategy trades against such cross-predictability. The negative cross-covariance term more than offsets the positive contribution of the cross-sectional variation in means (0.53% per year).

Whereas the cross-sectional strategy's weights are based on the factors' *relative* performance, those of the time-series strategy are based on the factors' *absolute* performance. The time-series strategy is therefore a pure bet on factor autocorrelations; in principle, this strategy could be long or short all factors at the same time whereas the cross-sectional strategy is always a balanced mix of long and short positions. The weight on factor f in month t is now just its return over the formation period, $w_t^f = r_{-t}^f$. Following Lewellen (2002) and Moskowitz et al. (2012), the time-series momentum strategy's expected return can be decomposed as:

$$\mathrm{E}[\pi_t^{\mathrm{TS}}] = \frac{1}{F}\mathrm{E}\Big[\sum_{f=1}^{F} r_{-t}^f r_t^f\Big] = \frac{1}{F}\sum_{f=1}^{F}\left[\mathrm{cov}(r_{-t}^f, r_t^f) + (\mu^f)^2\right] = \frac{1}{F}\mathrm{Tr}(\Omega) + \frac{1}{F}\sum_{f=1}^{F}(\mu^f)^2, \qquad (5)$$

where the definitions are the same as those in equation (3). Equation (5) shows that the time-series momentum profits stem either from autocorrelation in factor returns or from mean returns that are either very positive or negative.

Table 4 shows that, over the 1963–2015 sample period, the monthly premium of the time-series strategy is 4.88% with a t-value of 4.65. The decomposition of these profits into the autocorrelation and mean-squared components shows that this premium largely derives from the autocorrelation in factor returns; the annualized premiums associated with these two components are 3.01% (t-value of 2.61) and 1.88% with (t-value = 4.49). The time-series strategy outperform the cross-sectional strategy because it does not bet on factors exhibiting negative cross-covariance; it is a purer bet on the autocorrelation in factor returns.

4 Factor momentum and individual stock momentum

In this section we show that if stock returns obey a factor structure, then factor momentum transmits into the cross section of stock returns and takes the form of *cross-sectional* stock momentum of Jegadeesh and Titman (1993). We first show that factor momentum can enter the cross section through two different channels. We then show that all of cross-sectional stock momentum emanates from factor momentum.

4.1 Transmission of factor momentum into the cross section of stock returns: Framework

In multifactor models of asset returns, such as the Intertemporal CAPM of Merton (1973) and the Arbitrage Pricing Theory of Ross (1976), several sources of risk determine and alter expected returns. Consider a linear factor model in which asset excess returns obey an F-factor structure,

$$R_{s,t} = \sum_{f=1}^{F} \beta_s^f r_t^f + \varepsilon_{s,t}, \tag{6}$$

where t denotes time, R_s is stock s's excess return, r^f is the return on factor f, β_s^f is stock s's beta on factor f, and ε_s is the stock-specific return component that should not command a risk premium in the absence of arbitrage. We assume that the factors and the stock-specific return components do not exhibit any lead-lag relationships, that is, $\mathrm{E}[r_{t'}^f \varepsilon_{s,t}] = 0$.

We now assume that asset prices evolve according to equation (6) and examine the payoffs to a cross-sectional momentum strategy; this strategy, as before, chooses weights that are proportional to stocks' performance relative to the cross-sectional average. The expected payoff to the position in stock s is

$$\mathrm{E}[\pi_{s,t}^{\mathrm{mom}}] = \mathrm{E}\big[(R_{s,-t} - \bar{R}_{-t})(R_{s,t} - \bar{R}_t)\big], \tag{7}$$

where $\bar{R}$ is the return on an equal-weighted index. Under the return process of equation (6), this expected profit becomes

$$\begin{aligned} \mathrm{E}[\pi_{s,t}^{\mathrm{mom}}] \;=\; & \sum_{f=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^f)\,(\beta_s^f - \bar{\beta}^f)^2\big] + \sum_{\substack{f=1 \\ f\neq g}}^{F} \sum_{g=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^g)\,(\beta_s^g - \bar{\beta}^g)\,(\beta_s^f - \bar{\beta}^f)\big] \\ & + \mathrm{cov}(\varepsilon_{s,-t}, \varepsilon_{s,t}) + (\eta_s - \bar{\eta})^2, \end{aligned} \tag{8}$$

where η_s is stock s's unconditional expected return. The expectation of equation (8) over the cross section of N stocks gives the expected return on the cross-sectional momentum strategy,

$$\begin{aligned} \mathrm{E}[\pi_t^{\mathrm{mom}}] \;=\; & \underbrace{\sum_{f=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^f)\,\sigma_{\beta_f}^2\big]}_{\text{autocovariances}} + \underbrace{\sum_{\substack{f=1 \\ f\neq g}}^{F} \sum_{g=1}^{F} \big[\mathrm{cov}(r_{-t}^f, r_t^g)\,\mathrm{cov}(\beta^g, \beta^f)\big]}_{\text{cross-covariances}} \\ & + \underbrace{\frac{1}{N}\sum_{s=1}^{N} \big[\mathrm{cov}(\varepsilon_{-t}^s, \varepsilon_t^s)\big]}_{\substack{\text{autocorrelation} \\ \text{in residuals}}} + \underbrace{\sigma_\eta^2}_{\substack{\text{variation in} \\ \text{mean returns}}}, \end{aligned} \tag{9}$$

where N is the number of stocks and $\sigma_{\beta_f}^2$ and σ_η^2 are the cross-sectional variances of the portfolio loadings and unconditional expected returns.

Equation (9) shows that the profits of the cross-sectional stock momentum strategy can emanate from four sources:

1. Positive autocorrelation in factor returns induces momentum profits through the first term. Cross-sectional variation in betas amplifies this effect.

2. The lead-lag return relationships between factors could also contribute to stock momentum profits. The strength of this effect depends both on the cross-serial covariance in factor returns and the covariance in stocks' factor loadings. This condition is restrictive: the cross-serial correlation of returns and the covariances of betas have to have the same signs. It would need to be, for example, that (1) SMB return in period 1 positively predicts HML returns in period 2 and (2) that SMB and HML loadings also positively correlate.[6]

3. Autocorrelation in stocks' residual returns can also contribute to the profitability of the cross-sectional momentum strategy.

4. The cross-sectional variation in mean returns of individual securities can also contribute to momentum profits. If stocks' past returns provide good measures of their unconditional means, a cross-sectional momentum is long stocks with high mean returns and short those with low means (Conrad and Kaul, 1998).

Does factor momentum contribute to the returns of cross-sectional momentum strategies? We focus on the role of the first term of equation (9); this is the term through which the autocorrelation in factor returns could add to the profits of cross-sectional momentum strategies. If the autocorrelation in factor returns contributes to the profits of these strategies, then these strategies should be more profitable when the "realized" autocorrelation in factor returns is positive.

[6] In Appendix A.2 we show, under the Fama-French five-factor model, this term is negligible relative to the autocovariance term because of this joint condition.

Table 5: Unconditional and conditional correlations with the equity momentum factor (UMD)

This table reports correlations between UMD and factor returns: ρ is UMD's unconditional correlation with the raw factor; ρ^+ is the correlation with the factor conditional on the factor's return over the prior year being positive; and ρ^- is the correlation conditional on the prior-year return being negative. The first row shows the estimates for a diversified factor which is defined as the average of the 20 factors. The conditional correlations on this row are computed by averaging factors with positive or negative returns over the prior year. The last two columns report statistics for the test that the conditional correlations are equal, H_0: $\rho^+ = \rho^-$. This test uses Fisher's (1915) z-transformation (the inverse hyperbolic tangent transformation), $\sqrt{n-3}\left(\tanh^{-1}(\hat{\rho}) - \tanh^{-1}(\rho)\right) \sim N(0,1)$, where $\tanh^{-1}(\rho) = \frac{1}{2}\frac{\ln(1+\rho)}{\ln(1-\rho)}$.

	Unconditional correlation	Conditional correlations		Test: H_0: $\hat{\rho}^+ = \hat{\rho}^-$	
Anomaly	$\hat{\rho}$	$\hat{\rho}^+$	$\hat{\rho}^-$	z-value	p-value
Diversified	0.05	0.44	−0.50	17.87	0.00
			U.S. factors		
Size	−0.02	0.16	−0.39	6.99	0.00
Value	−0.17	0.23	−0.57	10.55	0.00
Profitability	0.09	0.44	−0.39	10.50	0.00
Investment	−0.01	0.19	−0.35	6.53	0.00
Accruals	0.07	0.24	−0.18	5.04	0.00
Betting against beta	0.17	0.36	−0.13	5.18	0.00
Cash-flow to price	−0.06	0.18	−0.40	7.35	0.00
Earnings to price	−0.14	0.16	−0.55	9.21	0.00
Liquidity	−0.02	0.07	−0.16	2.63	0.01
Long-term reversals	−0.06	0.11	−0.41	6.46	0.00
Net share issues	0.13	0.36	−0.41	9.79	0.00
Quality minus junk	0.24	0.47	−0.41	11.12	0.00
Residual variance	0.20	0.67	−0.56	17.87	0.00
Short-term reversals	−0.29	−0.36	−0.23	−1.50	0.13
			Global factors		
Size	0.08	0.17	−0.02	1.63	0.10
Value	−0.16	0.17	−0.49	5.50	0.00
Profitability	0.26	0.33	−0.17	3.40	0.00
Investment	0.07	0.41	−0.47	7.79	0.00
Betting against beta	0.24	0.27	0.15	0.87	0.38
Quality minus junk	0.41	0.51	−0.14	5.17	0.00

We first examine the connection between factor momentum and individual stock momentum by measuring factors' correlations with Carhart's (1997) UMD factor. In Table 5 we report three correlations estimates for each factor: unconditional correlation, correlation conditional on the factor's return over the prior one-year period being positive, and correlation conditional on this return being negative. The idea is the following. If a factor's returns exhibit continuation *and* this continuation also adds to the profitability of stock momentum strategies, then factors should correlate more with UMD when the factor's past return is positive.

Table 5 shows that the unconditional correlations between the factors and UMD are low; 11 out of the 20 correlations with the individual factors are positive, and the correlation between UMD and diversified factor—which is the average of all 20 factors—is 0.05. The correlations conditional on past returns, however, are remarkably different. Except for the short-term reversals factor, factors correlate more with UMD when the factors past return is positive. For 17 of these 19 factors, the difference is statistically significant at the 5% level. On the first row, we define all factors into two groups each month based on their past returns. The estimates show that the basket of factors with positive past returns has a correlation of 0.44 with UMD; the basket of factors with negative returns has a correlation of −0.50.

The estimates in Table 5 suggests that factor momentum is a significant driver of the momentum in the cross section of stock returns. Almost all factors are positively autocorrelated (Table 2); these autocorrelations in factor returns *aggregate* into UMD. Put differently, UMD is not a distinct factor—it emerges only through the properties of the other factors. A strategy that trades cross-sectional stock momentum is an indirect method of timing the factors to profit from the autocorrelations in their returns.

4.2 Explaining the returns on portfolios sorted on equity momentum

The analysis in Table 5 suggests that the autocorrelation in factor returns may be an integral part of the profitability of cross-sectional momentum strategies. We now measure the connection between the profitability of these strategies and time-series factor momentum. The time-series factor momentum strategy is defined as above: it is long factors that have earned positive returns over the prior year and short those that have earned negative returns.

In Table 6 we compare the performance of three asset pricing models in pricing portfolios sorted by prior one-year returns skipping a month; the sorting variable is the same as that used to construct the UMD factor of Carhart (1997). The first model is the Fama-French five-factor model; the second model is this model augmented with the UMD factor; and the third model is the five-factor model augmented with the factor time-series momentum strategy. We report alphas for the deciles and, for the models 2 and 3, the factor loadings against UMD and factor time-series momentum.

The momentum effect remains evident in the alphas of the Fama-French five-factor model. The alpha for the loser portfolio is -0.78% per month (t-value = -4.06) and for the winner portfolio is 0.61% (t-value = 4.89). The average absolute alpha across the deciles is 27 basis points. By adding Carhart's momentum factor, we significantly improve the model's ability to price these portfolios. The average absolute monthly alpha falls to 13 basis points, and the profitability of the long-short portfolio falls from 1.4% to 0.3%. Yet, the alpha associated with the long-short portfolio is statistically significant with a t-value of 2.53. Because both the deciles and UMD sort on the same variable—prior one-year return skipping a month—the UMD slopes monotonically increase in deciles; while the slope estimate in the bottom decile is -0.92, that in the top decile is 0.58.

The model augmented with the time-series factor momentum strategy outperforms the Carhart (1997) model in pricing the momentum portfolios. The average absolute alpha falls to 12 basis points per month; the Gibbons et al. (1989) test statistic falls from 3.26 to 2.55; and the alpha of the high-minus low falls from 0.29% to 0.24% (t-value = 1.09). Similar to the Carhart (1997) model, the estimated slopes against the factor momentum strategy increase monotonically from

Table 6: Time-series regressions on momentum sorted portfolios

This table compares the performance of three asset pricing models in explaining the monthly excess returns on ten portfolios sorted by prior one-year returns skipping a month. The three models are: (1) the Fama-French five-factor model with the market, size, value, profitability, and investment factors; (2) the five-factor model augmented with Carhert's UMD factor; and (3) the five-factor model augmented with the time-series factor momentum strategy. The time-series factor momentum strategy is long the factors with positive prior one-year returns and short those with negative prior one-year returns. The 20 factors used in constructing this strategy are listed in Table 1. We report alphas for each of the three models and loadings against the UMD factor and the time-series factor momentum strategy. We compute Gibbons et al. (1989) test statistics using the returns on the decile portfolios. This test statistic is distributed as $F(N, T-N-1)$ under the null hypothesis that the alphas are jointly zero, where $N = 10$ is the number of test assets and $T = 618$ is the number of observations in the time series. The sample period starts in July 1964 and ends in December 2015.

	Asset pricing model				
	FF5	FF5 + UMD		FF5 + TSMOM	
Decile	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{b}_{\text{umd}}$	$\hat{\alpha}$	$\hat{b}_{\text{tsmom}}$
Low	−0.78	−0.10	−0.92	−0.05	−2.44
	(−4.06)	(−0.92)	(−36.32)	(−0.33)	(−19.46)
2	−0.34	0.17	−0.69	0.18	−1.76
	(−2.53)	(2.50)	(−45.18)	(1.70)	(−20.25)
3	−0.22	0.18	−0.54	0.18	−1.32
	(−1.92)	(2.92)	(−37.36)	(1.90)	(−17.37)
4	−0.16	0.08	−0.33	0.13	−0.96
	(−1.89)	(1.24)	(−21.97)	(1.74)	(−16.43)
5	−0.19	−0.06	−0.17	−0.05	−0.47
	(−2.74)	(−0.98)	(−11.88)	(−0.70)	(−8.84)
6	−0.16	−0.12	−0.06	−0.09	−0.23
	(−2.44)	(−1.79)	(−3.76)	(−1.38)	(−4.27)
7	−0.11	−0.16	0.07	−0.14	0.09
	(−1.72)	(−2.47)	(4.46)	(−2.09)	(1.76)
8	0.06	−0.10	0.22	−0.06	0.42
	(0.93)	(−1.71)	(16.57)	(−0.93)	(7.82)
9	0.10	−0.14	0.32	−0.10	0.65
	(1.17)	(−2.33)	(22.99)	(−1.28)	(10.47)
High	0.61	0.19	0.58	0.18	1.43
	(4.89)	(2.43)	(32.30)	(1.72)	(16.61)
High − Low	1.39	0.29	1.50	0.24	3.87
	(4.94)	(2.53)	(56.85)	(1.09)	(22.31)
Avg. $\lvert\hat{\alpha}\rvert$	0.27	0.13		0.12	
GRS F-value	4.43	3.26		2.55	
GRS p-value	0.00%	0.04%		0.50%	

bottom decile's −2.44 to top decile's 1.43.

The fact that the five-factor model augmented with factor momentum outperforms the model augmented with UMD is surprising. The UMD model sets a challenging standard because both the factor and the test assets sort on the same variable; that is, UMD targets momentum as directly as, say, HML targets portfolios sorted by book-to-market. Factor momentum's ability to explain individual stock momentum is consistent with Table 5's message: individual stock momentum is about factor momentum being transmitted into the cross section of stock returns.

4.3 Alternative momentum factors: Spanning tests

In Table 7 we show that, in addition to the "standard" individual stock momentum of Jegadeesh and Titman (1993), factor momentum fully subsumes the informativeness of many other cross-sectional momentum strategies. In addition to the UMD factor, which sorts by stocks' prior one-year returns skipping a month, we construct three other momentum factors using the same methodology: Industry-adjusted momentum of Cohen and Polk (1998) sorts stocks' by their industry-adjusted returns; intermediate momentum of Novy-Marx (2012) sorts stocks by their prior one-year returns, skipping *six months*; and Sharpe ratio momentum of Rachev et al. (2007) sorts stocks by the returns scaled by the volatility of returns. We also construct the industry momentum strategy of Moskowitz and Grinblatt (1999). This strategy sorts 20 industries based on their prior six-month returns and and takes long positions in the top three industries and short positions in the bottom three industries.

Panel A of Table 7 introduces the alternative momentum factors alongside the time-series factor momentum strategy. Each factor earns statistically significant average returns and Fama-French five-factor model alphas. Although the average return associated with the time-series momentum strategy is the lowest—0.35% per month—it is also the least volatile by a significant margin. Its

Table 7: Alternative definitions of momentum: Spanning tests

Panel A reports monthly average returns and Fama-French five-factor model alphas for alternative momentum factors. Every factor, except for industry momentum, is similar to the UMD factor of Jegadeesh and Titman (1993) ("standard momentum"). We sort stocks into six portfolios by market values of equity and prior performance. A momentum factor's return is the average return on the two high portfolios minus that on the two low portfolios. Industry momentum uses the Moskowitz and Grinblatt (1999) methodology; it is long the top three industries based on prior six-month returns and short the bottom three industries, where each stock is classified into one of 20 industries following (Moskowitz and Grinblatt, 1999, Table I). Panel A also reports references for the original studies that use these alternative definitions. Panel B reports estimates from spanning regressions in which the dependent variable is the monthly return on either one of the momentum factors or factor momentum. When the dependent variable is one of the momentum factors, we estimate regressions that augment the five-factor model with factor momentum. We report the intercepts and the slopes for factor momentum. When the dependent variable is factor momentum, we estimate regressions that augment the five-factor model with one of the momentum factors or, on the last row, with all five momentum factors, We report the intercepts and the slopes for the momentum factors. The sample begins in July 1964 and ends in December 2015.

Panel A: Factor means and Fama-French five-factor model alphas

		Monthly returns			FF5 model	
Momentum definition	Reference	$\bar{r}$	SD	$t(\bar{r})$	$\hat{\alpha}$	$t(\hat{\alpha})$
	Individual stock momentum					
Standard momentum	Jegadeesh and Titman (1993)	0.70	4.27	4.10	0.74	4.28
Ind.-adjusted momentum	Cohen and Polk (1998)	0.47	2.80	4.18	0.50	4.58
Industry momentum	Moskowitz and Grinblatt (1999)	0.35	4.13	2.09	0.39	2.26
Intermediate momentum	Novy-Marx (2012)	0.58	3.12	4.60	0.63	4.97
Sharpe ratio momentum	Rachev et al. (2007)	0.63	3.43	4.55	0.69	4.88
	Factor momentum					
Factor momentum		0.35	1.23	7.05	0.30	6.13

Sharpe and information ratio, which are proportional to the t-values associated with the average returns and five-factor model alphas, are therefore the highest among all the factors.

The first two columns of Panel B show estimates from spanning regressions in which the dependent variable is one of the momentum factors. The model is the Fama-French five factor model augmented with factor momentum. These regressions can be interpreted both from the investment and asset pricing perspectives. From an investment perspective, a statistically significant alpha

Panel B: Spanning regressions

Individual stock momentum definition, SMOM	Dependent variable = Individual stock momentum $\hat{\alpha}$	Individual stock momentum FMOM	Factor momentum $\hat{\alpha}$	Factor momentum SMOM
Standard momentum	0.01 (0.07)	2.43 (23.49)	0.16 (4.32)	0.20 (23.49)
Industry-adjusted momentum	0.11 (1.18)	1.32 (17.95)	0.17 (4.17)	0.26 (17.95)
Industry momentum	−0.22 (−1.48)	2.02 (17.29)	0.24 (5.87)	0.16 (17.29)
Intermediate momentum	0.17 (1.64)	1.52 (17.92)	0.16 (3.89)	0.23 (17.92)
Sharpe ratio momentum	0.09 (0.83)	2.00 (24.02)	0.13 (3.73)	0.24 (24.02)
All of above			0.14 (3.96)	.[†]

[†]Note: This regression includes all six individual stock momentum factors as explanatory variables in addition to the five factors of the Fama-French five-factor model: standard momentum, industry-adjusted momentum, industry momentum, intermediate momentum, and Sharpe ratio momentum.

implies that an investor would have earned a higher Sharpe ratio by having traded the left-hand side factor in addition to the right-hand side factors (Huberman and Kandel, 1987). From an asset pricing perspective, a statistically significant alpha implies that the asset pricing model that only contains the right-hand side variables is dominated by a model that also contains the left-hand side factor (Barillas and Shanken, 2017).

Although all definitions of momentum earn statistically significant average returns and five-factor model alphas, factor momentum spans most of them. Consistent with Table 6, for example, time-series factor momentum leaves standard momentum (UMD) with an alpha of 0.01% per month (t-value = 0.07). Table 7 shows that factor momentum also spans the other four forms of momentum: industry-adjusted momentum, industry momentum, intermediate momentum, and Sharpe ratio.

The last two columns of Table 7 show that none of the alternative definitions of momentum span time-series factor momentum. Across all six specifications reported in this panel, the lowest t-value for the alpha earned by the factor momentum is 3.73. The last specification augments the Fama-French five-factor model with all five momentum factors. In this specification factor momentum's alpha is significant with a t-value of 3.96. Table 7 indicates that factor momentum contains information not present in any other forms of momentum and yet, at the same time, no other form of momentum is at all informative about the cross section of stock returns when controlling for factor momentum.

4.4 Individual stock momentum versus factor momentum with alternative sets of factors

The factor momentum strategy takes positions in up to 20 factors. Tables 6 and 7 show that this version of factor momentum explains individual stock momentum. In Figure fig:combinations we measure the extent to which this result is sensitive to the number and identity of the factors included in factor momentum.

In this figure we construct random combinations of factors, ranging from one factor to the full set of 20 factors. We then construct a factor momentum strategy that trades this particular set of factors and estimate two regressions. The first regression is the Fama-French five-factor model and the dependent variable is the factor momentum strategy. The dependent variable in the second regression is UMD and the model is the is the Fama-French five-factor model augmented with factor momentum. We draw 20,000 random combinations of factors for each set size, record the t-values associated with the alphas from these models, and then plot averages of these t-values as a function of the number of factors. In Figure 3 we also show, for reference, the t-value associated with UMD's alpha in the five-factor model.

Figure 3 shows that the t-value associated with factor momentum's five-factor model alpha

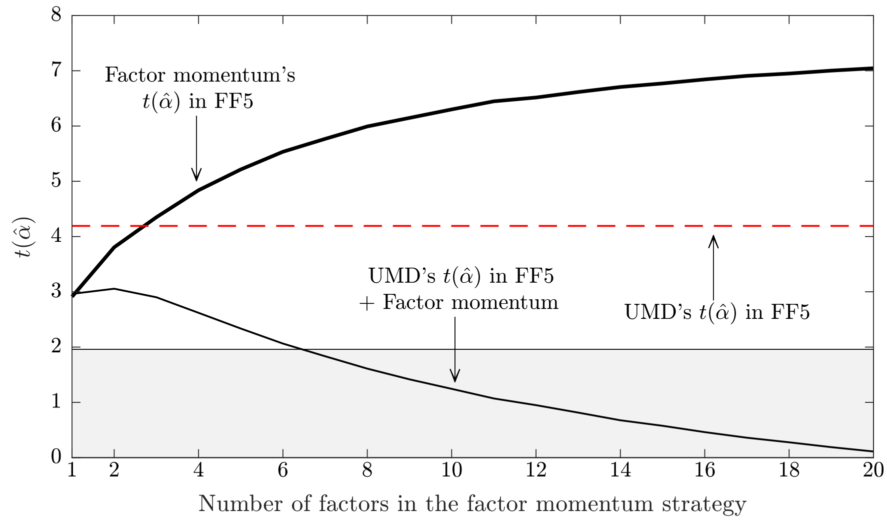

Figure 3: **Individual stock momentum versus factor momentum as a function of the number of factors.** We form random subsets of the 20 non-momentum factors listed in Table 1 and form time-series factor momentum strategies that trade these factors. A time-series factor momentum strategy is long factors with positive returns over the prior one-year period and short those with negative returns. In this figure the number of factors ranges from 1 to 20. The thick line represents the factor momentum strategy's average $t(\hat{\alpha})$ from the Fama-French five-factor model regression; the thin line represents UMD's average $t(\hat{\alpha})$ from a regression that augments the five-factor model with the factor momentum strategy; and the dashed line denotes UMD's $t(\hat{\alpha})$ from the Fama-French five-factor model regression. The shaded region indicates t-values below 1.96.

monotonically increases in the number of factors. When the strategy is alternates between long and short positions in just factor, the average t-value is 2.91; when it trades 10 factors, it is 6.30; and when we reach 20 factors, it is 7.05. At the same time, factor momentum's ability to span UMD improves. The typical one-factor factor momentum strategy leaves UMD with an alpha that is statistically significant with a t-value of 2.96. However, when the number of factors has increases to 10, this average t-value has decreased to 1.24; and with all 20 factors, this t-value is 0.11. These estimates suggest that factor momentum's ability to span UMD is not specific to the set of factors we consider; as the number of factors increases, factor momentum found in most sets of factors explain individual stock momentum.

4.5 An analysis of momentum crashes

Individual stock momentum sometimes crashes (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016). If the profits of momentum strategies emanate from factor autocorrelations, then momentum crashes should stem from *changes* in these autocorrelations. This is an additional testable prediction of our proposition that factor momentum drives individual stock momentum. We test for this connection by creating a proxy of the first term in equation (9) for the average factor. We can rewrite this term as a function of factor autocorrelations:[7]

$$\sum_{f=1}^{F} \left[\text{cov}(r_{-t}^{f}, r_{t}^{f})\, \sigma_{\beta_f}^{2}\right] \approx \frac{1}{12} \sum_{f=1}^{F} \left[\rho_{\text{auto}}^{f}\, \sigma_{\rho_{s,f} \times \sigma_s}^{2}\right], \tag{10}$$

where ρ_{auto}^{f} is the average autocorrelation between factor f's return in month t and its average return from month $t-12$ to $t-1$ and $\sigma_{\rho_{s,f} \times \sigma_s}^{2}$ is the cross-sectional variance of stock s's correlation with factor f ($\rho_{s,f}$) multiplied its volatility (σ_s). If the cross-sectional dispersion in the correlation between factors and individual stocks do not vary much across factors, momentum profits directly relate to the summation of factor autocorrelations.

We create an aggregate factor autocorrelation index to proxy for the term in equation (10). We first define factor f's autocorrelation in month t as

$$\rho_{\text{auto},t} = \frac{r_{-t}\, r_t - \mu_{-t}\, \mu_t}{\sigma_{r_t}\, \sigma_{r_{-t}}} \approx \frac{r_{-t}\, r_t - \mu^2}{\sigma^2/\sqrt{12}}, \tag{11}$$

[7] Equation (10) can be derived as follows. Because $r_{-t} = \frac{1}{12}\left(r_{t-12} + r_{t-11} + \cdots + r_{t-1}\right)$, factor variance can be removed from the expression:

$$\begin{aligned}\sum_{f=1}^{F} \left[\text{cov}(r_{-t}^{f}, r_{t}^{f})\, \sigma_{\beta_f}^{2}\right] &= \sum_{f=1}^{F} \Big[\Big(\frac{1}{12} \sum_{k=1}^{12} \rho_{r_{t-k}, r_t}^{f}\, \sigma_f^2\Big) \frac{1}{N} \sum_{s=1}^{N} \Big(\frac{\rho_{s,f}\sigma_s}{\sigma_f} - \frac{\overline{\rho_{s,f}\sigma_s}}{\sigma_f}\Big)^2\Big] \\ &= \sum_{f=1}^{F} \Big[\Big(\frac{1}{12} \sum_{k=1}^{12} \rho_{r_{t-k}, r_t}^{f}\Big) \frac{1}{N} \sum_{s=1}^{N} \big(\rho_{s,f}\sigma_s - \overline{\rho_{s,f}\sigma_s}\big)^2\Big].\end{aligned}$$

We get the presentation in equation (10) by denoting the summation of the autocorrelations from lags 1 to 12 by ρ_{auto}^{f}.

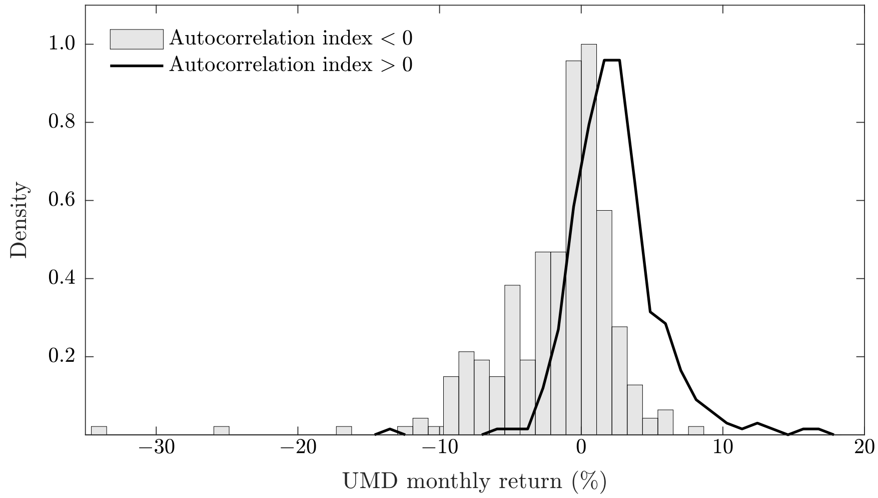

Figure 4: **Distribution of UMD returns conditional on factor autocorrelation.** A factor's autocorrelation in month t is computed as $\frac{r_{-t}\, r_t - \mu^2}{\sigma^2/\sqrt{12}}$, where μ and σ are the factor's mean and standard deviation and the formation period $-t$ runs from month $t-12$ to $t-1$. The aggregate factor autocorrelation index in month t is the cross-sectional average of these autocorrelations. This figure shows the distributions of UMD's monthly returns from July 1964 through December 2015 conditional on the aggregate autocorrelation index being negative (bars; $N = 264$) or positive (solid line; $N = 354$).

where μ and σ are the factor's mean and standard deviation over the sample period. The aggregate factor autocorrelation index in month t is the cross-sectional average of these autocorrelations. A positive autocorrelation index in month t indicates that the average factor in month t moved in the same direction as its return during the past year.

In Figure 4 we divide the sample into two regimes based on the sign of the autocorrelation index and then draw UMD's return distribution conditional on the regime. In the positive-autocorrelation regime, in which the average factor continues to move in the same direction as it did over the prior year, UMD's returns are typically positive. In these month's UMD's average return is 2.4% and its volatility is 3.3%. In the negative-autocorrelation regime, in which the average factor "turns" against its own past, UMD earns an average return of −1.6% with a standard deviation of 4.4%.[8]

The connection between the sign of the autocorrelation index and UMD is not limited to the

[8] In Appendix A.3 we report the mean, standard deviation, skewness, kurtosis, and the percentiles of UMD's return distribution conditional on the regime.

first two moments. Figure 4 shows that almost all UMD's left tail—its crashes—occur at times the autocorrelation index is negative. The 5th percentile of UMD's return in the positive regime is −1.9%; in the negative regime it is −8.8%.[9]

These estimates support the proposition that factor momentum emerges as momentum in the cross section of stock returns. Factor momentum explains all of UMD's returns unconditionally and the changes in the properties of factor momentum explain when momentum is likely to crash. When factor momentum ceases, the resulting reversal in factor returns feeds into stock returns and crashes individual stock momentum.

4.6 Momentum is not a distinct risk factor

Factor momentum's ability to span individual stock momentum, but not vice versa, suggests that individual stock momentum is a manifestation of factor momentum. If so, momentum is not a distinct risk factor. An investor who trades individual stock momentum is indirectly timing factors; a strategy that directly times factors outperforms this indirect method.

An implication of the connection between factor momentum and individual stock momentum is that the diversification benefits of momentum are more elusive than what the unconditional correlations might suggest. Consider, for example, the interaction between value and momentum. Table 5 shows that UMD's unconditional correlation with the U.S. HML is −0.17 and that with the global HML is −0.16. These negative correlations are consistent with the findings of Asness et al. (2013). This same table, however, shows that these correlations vary greatly depending on HML's performance over the prior year. In the U.S., the correlation is 0.22 following a year in which HML earned a positive return and −0.57 following a negative year. For the global HML,

[9] In Appendix A.4 we show that the state of factor autocorrelations significantly correlates with momentum crashes and "booms." At the individual factor level, the autocorrelations of 11 factors significantly increase the likelihood of a crash in a probit model. At the aggregate level, a one unit increase in the autocorrelation index lowers the probability of a momentum crash by 15% (z-value = −6.78).

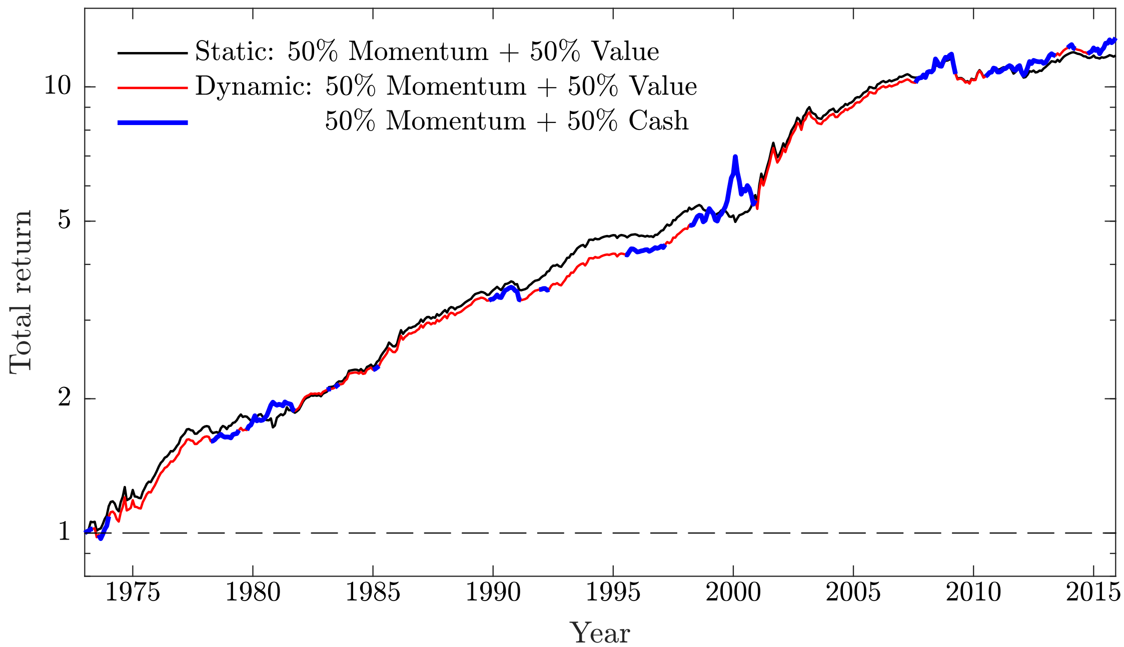

Figure 5: **Returns to diversifying momentum with value versus cash.** This figure shows total cumulative returns to two strategies that invest in momentum, value, and cash. The static strategy always invests 50% in momentum and 50% in value. The dynamic strategy invests 50% in momentum and 50% in value if value's return over the prior one-year period is positive and 50% in momentum and 50% in cash if its return is negative. Because value and momentum are zero-investment portfolios, the return on cash is set to zero. The dynamic strategy is drawn in red when the strategy is momentum-value and in blue when it is momentum-cash.

these correlations are 0.38 and −0.48. These switching signs matter because HML's return also depends on its prior returns—this is the essence of factor momentum! The U.S. HML earns an average monthly return of 0.15% after a negative year and a return of 0.43% after a positive year. For the global HML, these averages are −0.04% and 0.59% (Table 2).

Because of this interaction between individual stock momentum, value, and the autocorrelation in HML's returns, the diversification benefits of mixing momentum and value are limited. Figure 5 illustrates this issue by considering the 50-50 value/momentum strategy of Asness et al. (2013) and an alternative version of this strategy that sometimes replaces value with cash.[10] If HML has earned a positive annual return up to month t, this strategy is the 50-50 value/momentum

[10]In this analysis we use the same value factor as that used in Asness et al. (2013). The difference between this and the Fama-French value factor is that Asness et al. (2013) divide book values of equity by the end-of-June rather than end-of-December market values of equity (Asness and Frazzini, 2013).

strategy. However, if HML's return has been negative, this strategy becomes cash/momentum. Here, because both momentum and value are zero-investment portfolios, we set cash's return to zero—this position would finance a purchase of T-bills with T-bills. Figure 5 shows that over the 1973–2015 sample period, there is no meaningful difference between the original and the modified strategy.

This result is not specific to value's correlation with momentum. Table 5 shows that almost all factors "diversify" momentum better after they have performed poorly. The same issue then applies to all factors: when a factor correlates negatively with momentum, it typically is also the state in which the factor does not earn a meaningful premium. These state-dependent correlations are the direct consequence of the nature of momentum—momentum is not something that exists *separate* from the other factors.

5 Investor sentiment and factor momentum

Stambaugh et al. (2012) shows that many return anomalies are stronger following high levels of sentiment, and that this effect originates from the anomalies' underperforming legs. They suggest that this set of findings is consistent with a mispricing interpretation for the anomalies: anomalies may be due to mispricing that is particularly persistent in the presence of short-sale restrictions.

Factor momentum may be a different manifestation of the same mechanism. If a segment of the market—such as growth stocks—becomes overpriced, this overpricing may take time to correct itself because of short-sale restrictions. That is, asset prices do not immediately jump back to the fundamental levels but, rather, they converge over time.

Following Stambaugh et al. (2012), we use take the residual from a regression of the monthly sentiment index of Baker and Wurgler (2006) against a set of macroeconomic variables as measure of investor sentiment. The intuition of this regression is that this residual represents optimism

Table 8: Factor returns conditional on sentiment and factor momentum

Table reports average returns on factors conditional on past returns and sentiment. The high-minus-low strategy corresponds to factor momentum, which takes long positions in factors with positive returns over the prior year and short in those with negative returns.

	Prior factor returns		
Sentiment	Losers	Winners	W − L
Low	−0.22	0.50	0.71
	(−2.10)	(6.11)	(4.79)
High	0.35	0.54	0.18
	(3.42)	(5.83)	(1.32)

and pessimism that is not justified the state of the macroeconomy. In Appendix A.5 we confirm the findings of Stambaugh et al. (2012) by estimating regressions that explain factor returns with both investor sentiment and past factor returns. Both sentiment and past factor returns positively predict factor returns, and neither variable subsumes the other.

In Table 8 we measure the interaction between investor sentiment and factor momentum. We assign month t into a high or low sentiment regime depending on whether month t's sentiment is above or below median; and, similar to the winner and loser portfolios in Table 3, we classify each factor as a winner or loser depending on the sign of its average return over the prior year. The average underperforming factor earns an a negative return of −0.22% (t-value = −2.1) in the low-sentiment environment; in the same environment, factors with positive returns over the prior year earn a premium of 0.50% (t-value = 6.11). The factor momentum strategy therefore earns a return of 0.71% in this low-sentiment environment; this return is significant with a t-value of 4.79. In the high-sentiment environment, however, factor momentum's average return is 0.18% with a t-value of just 1.32.

Although the returns on the prior winners are almost the same in the low and high sentiment regimes—they are 0.50% and 0.54%—the returns on the prior losers are not. In the high sentiment environment, the average return on prior losers is significantly higher than in the low sentiment environment; the average factor earns a return of 0.35% (t-value = 3.42). The results in Table 8 are consistent with a mispricing interpretation for factor momentum and, by extension, all manifestation of individual stock momentum. A mispricing may take time to build up and, because of short-sale constraints, an asset's value does not immediately snap back to its fundamental value when arbitrageurs enter. Factor momentum may stem from asset values *converging* towards fundamental values.

6 Conclusion

Positive autocorrelation is a pervasive feature of factor returns. A strategy that takes a long position in factors with positive returns over the prior year significantly outperforms a passive strategy that holds all factors in equal proportions. A strategy that holds the basket of factors that have lost money over the prior year earns an average return indistinguishable from zero.

We link factor momentum to the momentum in the cross section of stock returns by considering decomposing the momentum strategy profits under the assumption that asset returns follow a factor structure. We use this representation to show that the autocorrelations in factor returns transmit into the cross section of stock returns through the variation in stocks' factor loadings. We show that the other potential mechanism of transmission, which relates to cross-serial covariances in factor returns and cross-sectional covariances in factor loadings, is unlikely to contribute to these profits. Consistent with this decomposition, we show that factor momentum completely explains both the "standard" momentum of Jegadeesh and Titman (1993) and other forms of it as well: industry-adjusted momentum, industry momentum, intermediate momentum, and Sharpe momentum. By contrast, these momentum factors do not even jointly span factor momentum. Our results imply that momentum is not a distinct, standalone risk factor; rather, a momentum "factor" is the summation of the autocorrelations found in the factor returns. An investor who trades momentum is indirectly timing the factors. The profits and losses of this strategy therefore ultimate depend on whether the autocorrelations in factor returns are positive or negative.

An additional testable implication of the proposition that individual stock momentum stems from factor momentum relates to momentum crashes. If individual stock momentum is, in the end, about factors, then we should be able to attribute momentum *crashes* back to the factors as well. Indeed, we find that momentum crashes when factors' autocorrelations turn negative.

Our results may be useful in guiding future research. A theory of momentum would need to explain not only why factor returns are typically positively autocorrelated—except when almost all of these autocorrelations, at the same time, turn negative.

Our results imply that individual stock momentum is unlikely about about firm-specific news—most factors are so well diversified that they should wash out most firm-specific information. Our results on the connection between factor momentum and investor sentiment suggest that the autocorrelation in factor returns, and, by extension, individual stock momentum, may stem from mispricing. Factors themselves may, in part, be about mispricing (Kozak et al., 2018; Stambaugh et al., 2012). If so, factor returns may positively autocorrelate because mispricings themselves mean-revert: asset prices first drift away from fundamental values as the mispricings emerge; and they must later drift towards fundamental values as arbitrageurs begin to profit from the mispricings.

References

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Asness, Cliff, and Andrea Frazzini, 2013, The devil in HML's details, *Journal of Portfolio Management* 39, 49–68.

Asness, Clifford S., Andrea Frazzini, and Lasse Heje Pedersen, 2017, Quality minus junk, AQR working paper.

Asness, Clifford S., Tobias J. Moskowitz, and Lasse Heje Pedersen, 2013, Value and momentum everywhere, *Journal of Finance* 68, 929–985.

Avramov, Doron, Si Cheng, Amnon Schreiber, and Koby Shemer, 2017, Scaling up market anomalies, *Journal of Investing* 26, 89–105.

Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov, 2013, Anomalies and financial distress, *Journal of Financial Economics* 108, 139–159.

Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *Journal of Finance* 61, 1645–1680.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Baquero, Guillermo, Jenke Ter Horst, and Marno Verbeek, 2005, Survival, look-ahead bias, and persistence in hedge fund performance, *Journal of Financial and Quantitative Analysis* 40, 493–517.

Barillas, Francisco, and Jay Shanken, 2017, Which alpha? *Review of Financial Studies* 30, 1316–1338.

Barroso, Pedro, and Pedro Santa-Clara, 2015, Momentum has its moments, *Journal of Financial*

Economics 116, 111–120.

Basu, Sanjoy, 1983, The relationship between earnings' yield, market value and return for nyse common stocks: Further evidence, *Journal of Financial Economics* 12, 129–156.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Beyhaghi, Mehdi, and Sina Ehsani, 2017, The cross-section of expected returns in the secondary corporate loan market, *Review of Asset Pricing Studies* 7, 243–277.

Bhojraj, Sanjeev, and Bhaskaran Swaminathan, 2006, Macromomentum: Returns predictability in international equity indices, *Journal of Business* 79, 429–451.

Bondt, Werner F. M., and Richard Thaler, 1985, Does the stock market overreact? *Journal of Finance* 40, 793–805.

Boyson, Nicole M., 2008, Hedge fund performance persistence: a new approach, *Financial Analysts Journal* 64, 27–44.

Brown, Stephen J., and William N. Goetzmann, 1995, Performance persistence, *Journal of Finance* 50, 679–698.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Cohen, Randolph B., and Christopher K. Polk, 1998, The impact of industry factors in asset-pricing tests, Kellogg Graduate School of Management working paper.

Conrad, Jennifer, and Gautam Kaul, 1998, An anatomy of trading strategies, *Review of Financial Studies* 11, 489–519.

Cooper, Michael J., Roberto C. Gutierrez, and Allaudeen Hameed, 2004, Market states and momentum, *Journal of Finance* 59, 1345–1365.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under-and overreactions, *Journal of Finance* 53, 1839–1885.

Daniel, Kent, and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fisher, Ronald A., 1915, Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population, *Biometrika* 10, 507–521.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2012, Trading costs of asset pricing anomalies, Fama-Miller Working Paper 14-05.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Griffin, John M., Xiuqing Ji, and J. Spencer Martin, 2003, Momentum investing and business cycle risk: Evidence from pole to pole, *Journal of Finance* 58, 2515–2547.

Grinblatt, Mark, Sheridan Titman, and Russ Wermers, 1995, Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior, *American Economic Review* 85, 1088–1105.

Hendricks, Darryll, Jayendu Patel, and Richard Zeckhauser, 1993, Hot hands in mutual funds: Short-run persistence of relative performance, 1974–1988, *Journal of Finance* 48, 93–130.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143–2184.

Huberman, Gur, and Shmuel Kandel, 1987, Mean-variance spanning, *Journal of Finance* 42, 873–

888.

Jagannathan, Ravi, Alexey Malakhov, and Dmitry Novikov, 2010, Do hot hands exist among hedge fund managers? An empirical evaluation, *Journal of Finance* 65, 217–255.

Jegadeesh, Narasimhan, 1990, Evidence of predictable behavior of security returns, *Journal of Finance* 45, 881–898.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Johnson, Timothy C., 2002, Rational momentum effects, *Journal of Finance* 57, 585–608.

Jostova, Gergana, Stanislava Nikolova, Alexander Philipov, and Christof W. Stahel, 2013, Momentum in corporate bond returns, *Review of Financial Studies* 26, 1649–1693.

Korajczyk, Robert A., and Ronnie Sadka, 2004, Are momentum profits robust to trading costs? *Journal of Finance* 59, 1039–1082.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2018, Interpreting factor models, *Journal of Finance* 73, 1183–1223.

Lee, Jongsub, Andy Naranjo, and Stace Sirmans, 2014, CDS momentum: Slow moving credit ratings and cross-market spillovers, Working paper.

Lesmond, David A., Michael J. Schill, and Chunsheng Zhou, 2004, The illusory nature of momentum profits, *Journal of Financial Economics* 71, 349–380.

Lewellen, Jonathan, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533–564.

Lo, Andrew W., and A. Craig MacKinlay, 1990, When are contrarian profits due to stock market overreaction? *Review of Financial Studies* 3, 175–205.

Loughran, Tim, and Jay R. Ritter, 1995, The new issues puzzle, *Journal of Finance* 50, 23–51.

McLean, R. David, and Jeffrey Pontiff, 2016, Does academic research destroy stock return predictability? *Journal of Finance* 71, 5–32.

Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf, 2012, Carry trades and global foreign exchange volatility, *Journal of Finance* 67, 681–718.

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Miffre, Joëlle, and Georgios Rallis, 2007, Momentum strategies in commodity futures markets, *Journal of Banking and Finance* 31, 1863–1886.

Moskowitz, Tobias J., and Mark Grinblatt, 1999, Do industries explain momentum? *Journal of Finance* 54, 1249–1290.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Novy-Marx, Robert, 2012, Is momentum really momentum? *Journal of Financial Economics* 103, 429–453.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Pástor, Luboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Rachev, Svetlozar, Teo Jašić, Stoyan Stoyanov, and Frank J. Fabozzi, 2007, Momentum strategies based on reward–risk stock selection criteria, *Journal of Banking and Finance* 31, 2325–2346.

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, 1985, Persuasive evidence of market inefficiency, *Journal of Portfolio Management* 11, 9–16.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Sagi, Jacob S., and Mark S. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum, *Journal of Financial Economics* 84, 389–434.

Sloan, Richard, 1996, Do stock prices fully reflect information in accruals and cash flows about future earnings? *The Accounting Review* 71, 289–315.

Stambaugh, Robert F., Jianfeng Yu, and Yu Yuan, 2012, The short of it: Investor sentiment and anomalies, *Journal of Financial Economics* 104, 288–302.

Szakmary, Andrew C., Qian Shen, and Subhash C. Sharma, 2010, Trend-following trading strategies in commodity futures: A re-examination, *Journal of Banking and Finance* 34, 409–426.

Titman, Sheridan, KC John Wei, and Feixue Xie, 2004, Capital investments and stock returns, *Journal of Financial and Quantitative Analysis* 39, 677–700.

A Appendix

A.1 Cross-sectional and time-series factor momentum strategies with different formation and holding periods

In the main text we form both the cross-sectional and time-series factor momentum strategies using prior one-year returns and rebalance these strategies monthly. Table A1 examines the performance of these strategies with formation and holding periods ranging from one month to two years. When the holding period is longer than a month, we use the overlapping portfolio approach of Jegadeesh and Titman (1993) to correct the standard errors. In month t, the return on a strategy with a k-month formation period is computed as the average return across h portfolios, where h is the length of the holding period. We construct these h strategies every month between months $t-1$ and $t-h$. This approach produces a single time series for each formation period-holding period combination.

Panel A of Table A1 examines the performance of time-series factor momentum strategies strategies. The time-series strategy with the one-month formation and holding periods earns an average return of 35 basis points (t-value $= 6.61$). However, unlike the cross-sectional strategies, the time-series strategies typically remain profitable also with longer formation and holding periods. All time-series momentum strategies are the most profitable when held for one month; at longer holding periods, this strategy's performance deteriorates because it cannot immediately rebalance away from factors whose average returns turn negative.

Panel B of Table A1 shows that cross-sectional momentum performs the best with one-month formation and holding periods. This strategy's average return is 31 basis points per month with a t-value of 5.98—the largest among all cross-sectional strategies. The profits on this short-term strategy decay quickly: the return on a strategy with one-month formation and three-month holding

Table A1: Average returns of time-series and cross-sectional factor momentum strategies

This table reports annualized average returns and t-values for time-series and cross-sectional factor momentum strategies that trade the 20 non-momentum factors listed in Table 1. The time-series factor momentum strategy is long factors with positive returns over a formation period that ranges from one month to two years and short factors with negative returns. The cross-sectional momentum strategy is long factors that earned above-average returns relative to other factors over the same formation period and short factors with below-average returns. We let the rebalancing frequency range from one month to two years. When the holding period is longer than a month, we use the Jegadeesh and Titman (1993) approach correct standard errors for the overlapping holding periods returns.

Panel A: Time-series factor momentum

Holding	Formation period						Formation period					
period	1	3	6	12	18	24	1	3	6	12	18	24
	Average returns						***t*-values**					
1	0.35	0.29	0.33	0.36	0.28	0.28	6.61	5.43	6.44	6.77	5.47	5.60
3	0.06	0.13	0.22	0.28	0.26	0.22	1.25	2.38	4.40	5.26	5.33	4.51
6	0.13	0.08	0.22	0.22	0.21	0.19	2.63	1.70	4.75	4.75	4.40	3.90
12	0.18	0.10	0.12	0.09	0.10	0.09	3.82	2.26	2.79	2.10	2.24	2.07
18	0.08	0.03	0.03	0.06	0.06	0.10	1.78	0.75	0.74	1.38	1.48	2.50
24	0.06	0.04	0.04	0.11	0.12	0.15	1.47	1.10	1.09	2.82	3.20	4.01

Panel B: Cross-sectional factor momentum

Holding	Formation period						Formation period					
period	1	3	6	12	18	24	1	3	6	12	18	24
	Average returns						***t*-values**					
1	0.30	0.24	0.20	0.24	0.17	0.15	5.99	4.91	4.28	4.99	3.80	3.48
3	0.00	0.04	0.07	0.14	0.11	0.09	0.05	0.78	1.57	3.03	2.70	2.14
6	0.07	0.05	0.09	0.09	0.08	0.08	1.50	1.16	1.95	2.04	1.89	1.84
12	0.08	0.07	0.00	−0.04	−0.01	0.01	1.75	1.65	0.00	−0.87	−0.19	0.22
18	0.00	−0.03	−0.06	−0.02	−0.01	0.01	−0.05	−0.70	−1.56	−0.48	−0.36	0.30
24	0.03	0.00	0.00	0.02	0.03	0.02	0.83	−0.06	−0.12	0.59	0.72	0.53

period is small and insignificant. Some of the strategies with longer formation periods, although less profitable initially, earn statistically significant profits at the three- and six-month holding periods.

A.2 Measuring the effects of the auto- and cross-serial covariances on momentum profits under the Fama-French five-factor model

Section 4.1 shows that the covariance structure of factor returns can induce momentum into the cross section of stocks through autocovariances and cross-serial correlations. Because factor returns are positively autocorrelated, the autocovariance component positively adds to the momentum profits. The effect of the cross-serial correlations, however, depends on the cross-serial covariance in factor returns and the covariance in stocks' factor loadings. This channel adds to momentum profits only if the cross-serial correlation of returns and the covariances of betas have the same signs. In this Appendix we show that, because of the restrictiveness of this condition, it is unlikely to matter in the data. We use the Fama-French five-factor model as an illustration.

Our approach requires estimating factor betas to measure the cross-sectional variances in factor betas and covariances between betas of different factors. For each stock at time t, we estimate factor betas by estimating rolling regressions that use one year of daily return data:

$$r_{s,d} = \alpha_s + \beta_{s,d}^{\text{MKTRF}}\text{MKTRF} + \beta_{s,d}^{\text{CMA}}\text{CMA} + \beta_{s,d}^{\text{HML}}\text{HML} + \beta_{s,d}^{\text{RMW}}\text{RMW} + \beta_{s,d}^{\text{SMB}}\text{SMB} + \epsilon_s. \quad (12)$$

We winsorize the beta estimates every month at the 1st and 99th percentiles to mitigate the effect of outliers. The top panel of Table A2 shows the autocovariances (elements on the diagonal of the matrix, boldfaced) and cross-serial covariances in factor returns. Most factors exhibit positive lead-lag relationship with other factors with the exception of the market. High returns on the market between $t-12$ and $t-1$ signal lower returns on the other factors at time t. Similarly, high returns on the size (SMB), profitability (RMW), and investment (CMA) factors are associated with lower future returns on the market.

The lower panel of Table A2 reports the average cross-sectional variances and covariance of betas that are estimated each month. With the exception of the investment and value factors, whose betas tend to be negatively correlated, the pairwise covariances are positive. We use the estimates in the two panels to measure the contributions of the auto-covariances and cross-serial covariances in factor returns to the profitability of the individual stock momentum strategy. Multiplying each

Table A2: Equity momentum profits due to the covariance structure of factor returns under the Fama-French five-factor model.

This table reports estimates of the terms in equation (9) under the assumption that the Fama-French five-factor model governs asset returns. The first panel shows the estimates of the auto- and cross-serial covariances between factor returns, $\left[\mathrm{cov}(r_{-t}^f, r_t^f)\right]$. The second panel shows the covariance matrix of factor betas $\left[\sigma_{\beta_f}^2 \text{ and } \mathrm{cov}(\beta^g, \beta^f)\right]$. At the bottom of the table we report the net total effects of the autocovariance-related terms, $\left[\sum_{f=1}^F \mathrm{cov}(r_{-t}^f, r_t^f)\sigma_{\beta_f}^2\right]$, and the cross-covariance terms, $\left[\sum_{f=1}^F \sum_{\substack{g=1 \\ f\neq g}}^F \mathrm{cov}(r_{-t}^f, r_t^g)\mathrm{cov}(\beta^g, \beta^f)\right]$.

Factor auto- and cross-serial covariances					
	CMA_{-t}	HML_{-t}	RMW_{-t}	SMB_{-t}	MKTRF_{-t}
CMA_t	**0.12**	0.08	0.14	0.05	−0.08
HML_t	0.13	**0.23**	0.11	0.08	−0.22
RMW_t	−0.02	−0.03	**0.17**	−0.01	−0.06
SMB_t	0.18	0.36	0.16	**0.29**	−0.29
MKTRF_t	−0.08	0.08	−0.07	−0.10	**0.14**
Covariance matrix of factor betas					
	β_{CMA}	β_{HML}	β_{RMW}	β_{SMB}	β_{MKTRF}
β_{CMA}	**2.52**	−0.86	0.29	0.16	0.11
β_{HML}	−0.86	**2.11**	0.75	0.26	0.34
β_{RMW}	0.29	0.75	**2.66**	0.45	0.14
β_{SMB}	0.16	0.26	0.45	**1.20**	0.47
β_{MKTRF}	0.11	0.34	0.14	0.47	**0.69**
Net auto-covariance, $\left[\sum_{f=1}^F \mathrm{cov}(r_{-t}^f, r_t^f)\sigma_{\beta_f}^2\right]$					1.69
Net cross-covariance, $\left[\sum_{f=1}^F \sum_{\substack{g=1 \\ f\neq g}}^F \mathrm{cov}(r_{-t}^f, r_t^g)\mathrm{cov}(\beta^g, \beta^f)\right]$					−0.13

cross-covariance of the top panel by its corresponding covariance in the lower panel and summing across all the pairs gives a small and negative estimate of −0.13% for the net effect of the cross-covariance component. In contrast, the contribution of autocovariance is significantly higher. Under the five-factor model, it is the autocorrelation in factor returns that turns into momentum in the cross section of stock returns.

Table A3: Distribution of momentum profits conditional on factor autocorrelation

This table reports the distributions of UMD returns conditional on the aggregate factor autocorrelation index. This index is the cross-sectional average of factors' autocorrelations (see equation (11)). We report the unconditional distributions of UMD and the distributions conditional on the autocorrelation being positive or negative. The sample begins in July 1964 and ends in December 2015.

		Aggregate factor autocorrelation index	
Statistic	Unconditional	< 0	> 0
Mean	0.70	−1.59	2.41
Standard deviation	4.27	4.40	3.26
Skewness	−1.37	−2.58	0.72
Kurtosis	13.59	16.95	7.66
Percentiles			
5th	−6.65	−8.81	−1.92
10th	−4.05	−6.91	−0.69
25th	−0.73	−3.16	0.36
50th	0.78	−0.48	2.07
75th	2.93	0.78	3.76
90th	4.99	2.54	5.93
95th	6.54	3.21	7.95
Number of months	618	264	354

A.3 Distribution characteristics of UMD as a function of the aggregate factor autocorrelation index.

Figure 4 in the text shows UMD's return distribution conditional on the sign of the aggregate factor autocorrelation index. Table A3 reports the first four moments and the percentiles of UMD's return distribution.

A.4 Momentum crash and booms probit regressions

Figure 4 suggests that individual stock momentum is likely to crash when the autocorrelations in factor returns turn negative. In this Appendix we measure the strength of this association. In Table A4 we reports estimates of probit models in which the dependent variable (Crash) is an indicator variable that takes the value of one when UMD's return is below its 10th percentile and zero otherwise:

$$Pr\left[\text{Crash}_t = 1 | \ \rho_{\text{auto},t}\right] = F(\alpha + \beta\rho_{\text{auto},t}), \tag{13}$$

where $\rho_{\text{auto},t}$ is the factor autocorrelation computed using equation (11), $Pr(\cdot|\cdot)$ denotes the conditional probability, and $F(\cdot)$ is the cumulative normal distribution. We also report estimates of another set of probit models in the dependent variable (Boom) takes the value of one if UMD's return is above its 90th percentile and zero otherwise. Because the probit model is nonlinear, we report the marginal effects implied by the slope estimates. In addition to the 20 non-momentum factors listed in Table **??**, we also add the market factor to the list of factors due to its role in generating momentum crashes (Daniel and Moskowitz, 2016).

Table A4 shows that the conditional probability of a momentum crash decreases in the autocorrelation across most factors. For the 11 factors with statistically significant estimates, a one-unit increase in the autocorrelation decreases the probability of a momentum crash between -1.09% (global investment) and -7.06% (for liquidity). The first row measures the effect of the aggregate autocorrelation index on the likelihood of a momentum crash. A one-unit increase in the index associates with a 15% lower probability of a crash (z-value $= -6.78$). This estimate exceeds that of all individual factors and it supports the view that equity momentum emerges as the summation of factor autocorrelations.

Just as momentum underperforms when factor returns are negatively autocorrelated, momentum returns are higher when autocorrelations intensify. The boom estimates are similar to the crash estimates but with the opposite signs. A one-unit increase in the aggregate autocorrelation index increases the probability of a "boom" by 15%. The aggregate autocorrelation index therefore has remarkably balanced effects on the crashes and booms: a one-standard deviation shock to aggregate autocorrelations has almost identical effects on the *probabilities* of booms and busts.

Table A4: Factor autocorrelation and momentum crashes and booms

This table reports estimates for probit regressions that measure the relationship between momentum crashes and booms and factor autocorrelations. Momentum crash is an indicator variable that takes the value of one if the realized UMD return lies below the distribution's 10th percentile and zero otherwise; momentum boom identifies UMD returns above the 90th percentile. Each row, except for the first one, measures the association between momentum crashes and booms and one of the factors. The independent variable in these regressions is the factor's autocorrelation in month t computed from equation (11). The independent variable on the first row is the aggregate factor autocorrelation index, which is defined as the cross-sectional average of factors' autocorrelations. This table reports the marginal effects associated with the autocorrelations, that is, the effect of a one-unit increase in autocorrelation on the likelihood of a crash or a boom in percentage points, the z-values associated with the slope estimates, their p-values, and McFadden's pseudo R^2s.

	Momentum crash			Momentum boom		
Factor	$\hat{\beta}$	z-value	R^2	$\hat{\beta}$	z-value	R^2
Aggregate autocorrelation index	−15.18	−6.78	38%	15.46	7.23	24%
U.S. factors						
Size	−3.89	−3.56	3%	3.38	3.91	4%
Value	−1.35	−1.88	1%	0.67	0.98	0%
Profitability	0.30	0.83	0%	0.06	0.20	0%
Investment	−2.33	−4.15	10%	1.91	4.02	13%
Market	−2.02	−1.78	1%	0.76	0.62	0%
Accruals	−2.46	−4.33	8%	−1.11	−2.39	3%
Betting against beta	−0.40	−0.62	0%	5.29	5.85	7%
Cash-flow to price	−2.51	−2.48	1%	1.67	1.88	1%
Earnings to price	−4.92	−4.18	4%	2.51	2.93	2%
Liquidity	−7.86	−5.62	7%	7.06	5.08	6%
Long-term reversals	−2.01	−2.47	2%	3.53	4.63	7%
Net share issues	−4.16	−4.98	11%	2.45	4.40	13%
Quality minus junk	−3.04	−4.75	11%	4.11	5.90	20%
Residual variance	−1.57	−1.55	0%	1.82	1.80	1%
Short-term reversals	−0.62	−0.75	0%	0.95	1.38	0%
Global factors						
Size	0.84	0.77	0%	1.58	1.94	4%
Value	0.19	0.39	0%	0.67	1.26	1%
Profitability	−0.87	−1.83	32%	0.38	1.39	24%
Investment	−0.33	−0.78	1%	0.31	0.74	1%
Betting against beta	−4.36	−4.02	12%	6.54	4.44	10%
Quality minus junk	−3.38	−3.58	34%	1.53	1.84	56%

A.5 Time-series regressions of factor returns on sentiment and lagged returns

We regress each factor's returns on lagged sentiment and past returns:

$$r_t = \alpha + \beta_s S_{t-1} + \beta_m r_{-t} + \epsilon_t, \tag{14}$$

where S_{t-1} is the sentiment index at time $t-1$ and r_{-t} is the average factor return from month $t-12$ to $t-1$. We also estimate alternative regressions in which replace both investor sentiment and prior average returns with indicator variables. The investor sentiment indicator variables takes the value of one when the sentiment is above the sample median and zero otherwise; the prior return indicator variables takes the value of one when prior average returns is positive and zero otherwise. This definition of the sentiment indicator variable is the same as that in Stambaugh et al. (2012).

Table A5 confirms the findings of Stambaugh et al. (2012). Anomaly returns are higher following periods of high sentiment, and often significantly so. The only statistically significant exception is the size factor. The first row reports the results for a pooled regression in which we cluster the standard errors by time. Both sentiment and momentum are statistically significant in the presence of each another. The right-hand side of Panel A shows that these conclusions do not change when we replace the continuous sentiment and prior factor return variables with indicator variables. In the pooled regression, high sentiment and momentum signal 0.27% and 0.49% higher factor returns next month with t-values of 2.75 and 4.19. Both sentiment and prior factor returns therefore independently predict factor returns.

Table A5: Time-series regressions of factor returns on sentiment and prior factor returns

Table report the coefficients from predictive time-series regressions of each factor returns on investor sentiment and momentum: $r_t = \alpha + \beta_s S_{t-1} + \beta_m r_{-t} + \epsilon_t$. The second five columns show the results from time-series regressions of factor returns on investor sentiment and momentum indicators: $r_t = a + b_s \mathbb{1}_{S_{t-1} > \text{median}} + b_m \mathbb{1}_{r_{-t} > 0} + \epsilon_t$. The sentiment indicator takes the value of one when investor sentiment is above the sample median and zero otherwise. The momentum indicator takes the value of one when the average return over the prior twelve months is positive and zero otherwise.

	Regression 1: Continuous variables				Regression 2: Indicator variables			
	Sentiment		Prior returns		Sentiment		Prior returns	
Factor	$\hat{b}_s$	$t(\hat{b}_s)$	$\hat{b}_m$	$t(\hat{b}_m)$	$\hat{b}_s$	$t(\hat{b}_s)$	$\hat{b}_m$	$t(\hat{b}_m)$
Pooled	0.14	2.00	0.25	2.56	0.26	2.75	0.49	4.19
				U.S. factors				
Size	−0.32	−2.50	0.22	1.70	−0.33	−1.30	0.63	2.41
Value	0.11	0.90	0.22	1.83	0.49	2.09	0.18	0.73
Profitability	0.29	3.17	0.17	1.41	0.47	2.73	0.31	1.70
Investment	0.12	1.48	0.21	1.77	0.26	1.61	0.22	1.28
Momentum	0.04	0.20	−0.01	−0.07	0.00	0.00	−0.10	−0.23
Accruals	−0.07	−0.82	−0.03	−0.18	−0.03	−0.16	0.14	0.88
Betting against beta	0.13	0.94	0.50	5.03	0.54	2.04	1.12	3.44
Cash-flow to price	−0.08	−0.62	0.16	1.24	0.05	0.22	0.28	1.12
Earnings to price	0.00	−0.03	0.21	1.85	0.34	1.44	0.23	0.92
Liquidity	0.16	1.05	0.08	0.52	0.19	0.63	0.41	1.31
Long-term reversals	0.05	0.53	0.38	3.20	0.02	0.12	0.71	3.41
Net share issues	0.37	3.51	0.20	1.68	0.41	2.13	0.07	0.33
Quality minus junk	0.45	4.26	0.07	0.56	0.50	2.52	0.47	2.25
Residual variance	0.95	4.23	0.01	0.10	1.16	2.73	0.85	2.00
Short-term reversals	−0.13	−0.99	−0.03	−0.17	0.10	0.38	0.21	0.70
				Global factors				
Size	−0.31	−1.53	0.04	0.19	−0.39	−1.58	0.20	0.81
Value	0.84	3.67	0.19	1.36	0.68	2.65	0.52	1.89
Profitability	0.13	0.98	0.22	1.21	0.13	0.76	0.34	1.63
Investment	0.60	3.41	0.20	1.50	0.50	2.36	0.53	2.44
Betting against beta	−0.01	−0.03	0.41	2.74	−0.11	−0.33	1.14	2.91
Quality minus junk	0.46	2.32	0.15	0.88	0.28	1.13	0.21	0.77
Momentum	0.54	1.56	−0.11	−0.61	0.03	0.07	0.30	0.54